RENORMALIZATION GROUP STUDY
OF THE MINIMAL SUPERSYMMETRIC
EXTENSION OF THE STANDARD MODEL
WITH SOFTLY BROKEN SUPERSYMMETRY

By

DIEGO J. CASTAÑO

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Diego J. Castaño

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Chairman: Pierre Ramond
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Using the renormalization group all the couplings of the Standard Model
and its minimal supersymmetric extension are run to two loops, taking full ac-
count of the Yukawa sector. It is found that in the standard model the gauge
couplings fail to unify, whereas extending the model through supersymmetry
achieves unification. Bounds are placed on the top quark mass by requiring
equality of the bottom and τ Yukawa couplings at the scale of unification. In its
simplest form, the supersymmetric model has a degenerate superparticle spec-
trum, and for $M_{SUSY} = 1$ TeV, and $M_b = 4.6$ GeV, one finds $139 \leq M_t \leq 194$
GeV, which remarkably satisfy the ρ–parameter bound. The corresponding
bounds on the Higgs mass are found to be $44 \leq M_H \leq 120$ GeV. The model is
then coupled to supergravity yielding a richer superparticle spectrum and mak-
ing it more accountable to experiment. This model has the attractive feature
that the electro-weak symmetry is radiatively broken. For the special case in
which global supersymmetry breaking arises solely from soft gaugino masses,
M_t is expected to be less than ~ 130 GeV. A higher upper bound is predicted,
if M_b is at the lower end of its experimental uncertainty.

CHAPTER 1
INTRODUCTION

In the last few years, it has become apparent, using the ever increasing accuracy in the measurement of the strong coupling, that supersymmetry (SUSY) affords an elegant means to achieve gauge coupling unification [1] at scales consistent with grand unified theories [2] (GUTs). Whereas in the standard model (SM) the three gauge couplings unify "two by two" forming the "GUT triangle," in the simplest minimal supersymmetric extension of the standard model (MSSM), these gauge couplings spectacularly unify at a point (within the experimental errors in their values). Given that the scale of unification in these models is generally above the lower bound set by proton decay, the so-called SUSY-GUTs have gained increasing interest. Constraints coming from Yukawa coupling unification in supersymmetric $SU(5)$ and $SO(10)$ models can be used to yield interesting predictions for various low energy parameters including the top quark mass [3,4].

This analysis employs the renormalization group (RG) [5] to extrapolate the parameters of the standard model and of its minimal supersymmetric extension to unexplored scales [6]. With the ever increasing precision of experiment, the inclusion of two loop effects is crucial. The complete Yukawa sector contribution is also included. In the standard model case, thresholds effects are implemented in both a simple and a more involved method. The results of these two methods are discussed. Numerical methods are used to evolve the parameters to different scales using the β functions found in the

literature [7], and the results are plotted for representative values of the Higgs boson and top quark masses. The running of the quark and lepton masses and of the Cabibbo-Kobayashi-Maskawa (CKM) angles is generally given through the running of the Yukawa matrices (even to one loop). In this work, the quark masses and CKM angles are evolved by diagonalizing the Yukawa matrices at every step in the Runge-Kutta method used in solving the β functions. Often it is assumed that the contribution of the Yukawa couplings matrix is given essentially by the top quark Yukawa since it is much larger than the others. Sometimes a better approximation is made by keeping only the diagonal entries. The present numerical technique represents a small improvement over these methods.

Chapter 2 consists of a basic introduction to renormalization and renormalization scheme dependence.

Chapter 3 addresses the standard model. A review of initial data extraction from experiments is presented. Many excellent reviews may be found in the literature, e.g., Marciano's [8] or Peccei's [9]. The determination of the standard model gauge couplings is discussed as is the initial data extraction of the Yukawas and the CKM angles. The extraction of the quark masses from data is also discussed. This is a complex issue well known to be marred by the nonperturbative nature of QCD. Hence, in the low energy regime, the pure QCD three-loop contribution is included in the analysis of the running of the quark masses. Initial data for lepton masses follow the quark discussion. Then the extraction of and constraints on the physical top and Higgs masses are considered. The scale dependence of the renormalized scalar vacuum expectation value is addressed. The method used to obtain the values of all running

parameters at the same initial scale is described in Appendix E. In the following section, threshold effects are discussed. Finally, a quantitative analysis of the results is presented. The effects of using one loop versus two loop β functions are contrasted and of including a proper versus a naive treatment of thresholds. Plots of all the running parameters over the entire range of mass scales are included and also used to display the effects discussed. Furthermore some tables are presented with actual numerical differences associated with these effects.

Chapter 4 addresses the minimal supersymmetric extension of the standard model. A brief discussion of supersymmetric models is presented. The procedure by which top quark mass bounds are determined using the equality of the bottom and tau masses at the scale of unification is discussed as well as the one light Higgs limit employed. The initial data for gauge couplings and quark masses are presented. This is followed by results on the top quark and Higgs boson masses. Finally, a discussion on how to improve the results is presented.

Chapter 5 implements some of the improvements discussed at the end of the last chapter. Namely, the supersymmetric two loop β functions are included. Also, soft symmetry breaking terms are added. These lead to a nondegenerate superparticle spectrum as well as to the radiative breaking of the electro-weak symmetry. Similar analyses have appeared in the literature but use one loop β functions and the tree level Higgs potential [10]. In Chapter 5, a brief discussion of the effective one loop potential is presented. The mass formulas for the sfermions, Higgses, charginos, and neutralinos are given. The boundary conditions at the unification scale in these minimal low energy supergravity models are discussed. Next the numerical procedure employed is described.

The treatment of thresholds and the "special" form of the β functions needed is discussed next. Finally, some preliminary results are presented.

Chapter 6 contains the conclusions of this work.

Appendix A contains all the needed two loop β functions for renormalization group studies of the standard model. Appendix C contains those of the minimal supersymmetric extension of the standard model. Many of these β functions have yet to appear in the literature in as general a form. Appendix B presents some examples in calculation of the β functions of Appendix C. Appendix D deals with issue of the vacuum expectation value's β function. A toy model is used to gather insight into the problem. The numerical solution routines used extensively throughout this work are discussed in Appendix E. Appendix F contains a cumbersome formula needed in the extraction of the Higgs boson physical mass. Finally Appendix G presents a novel use of the spinor representations of the orthosymplectic Lie supergroup $OSp(r/2m, R)$ [11].

CHAPTER 2
THE RENORMALIZATION GROUP

Renormalization is a reparametrization of a theory which renders Green functions and physical quantities finite order by order in perturbation theory. A specific choice of renormalized parameters defines a renormalization scheme. The physics is, of course, independent of how the theory is renormalized. A common way of relating bare and renormalized parameters is

$$g_o = g - \delta g \ , \tag{2.1}$$

where g_o is the bare parameter, g is the renormalized parameter, and δg is the counterterm. Fixing the counterterms by requiring them to consist only of the infinite terms needed to render the theory finite defines the minimal subtraction (MS) prescription [12]. A feature of the MS scheme is a mass scale μ which enters in the process of regularizing divergent integrals using dimensional regularization. Furthermore the unit of mass μ is used to keep couplings dimensionless when continuing to d dimensions in the dimensional regularization procedure. For example, if Eq. (2.1) represents any of the three gauge couplings of the Standard Model, then μ is introduced as follows to keep them dimensionless

$$g_o(\varrho\mu)^{-\epsilon} = g - \delta g \ , \tag{2.2}$$

where ϱ is a constant parametrizing the arbitrariness in the finite parts of divergent integrals in dimensional regularization and $\epsilon = (4 - d)/2$. Equation (2.2) defines a family of MS schemes. Choosing $\varrho = 1$ is the simplest MS scheme

which was described above. Choosing $\varrho^2 = e^{\gamma_E}/4\pi$, where $\gamma_E = 0.5722\ldots$ is the Euler-Mascheroni constant, defines the so-called modified minimal subtraction ($\overline{\text{MS}}$) prescription [13]. This scheme is the most commonly employed in QCD calculations, and it is the one adopted here. The free parameters of the Standard Model in the $\overline{\text{MS}}$ schemes are μ dependent. Their μ evolution is governed by the β functions of the renormalization group. Moreover, these running parameters are not in general equal to their corresponding physical values (consequently, for the masses, a convention is adopted wherein upper case M refers to physical values and lower case m denotes $\overline{\text{MS}}$ values). This is to be contrasted with the on-shell renormalization scheme in which, for example, the renormalized masses equal their physical values and the renormalized electromagnetic coupling equals the fine structure constant. However, the MS schemes have the attractive characteristic that the β functions are μ independent and therefore particularly simple to integrate. Physical quantities $P(\{g_i(\mu)\}, \mu)$ expressed in terms of μ and the running parameters of the theory, $\{g_i(\mu)\}$, must be μ independent

$$\mu \frac{d}{d\mu} P(\{g_i(\mu)\}, \mu) = (\mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial g_i})P = 0 \; , \tag{2.3}$$

where the β_i are the β functions. The two-loop β functions of the Standard Model have been collected in Appendix A.

As mentioned above physical quantities are renormalization scheme independent. However, this assumes that calculations can be done without approximation. In reality, calculations are only perturbative approximations and these do depend on the renormalization scheme.

Consider massless QCD with the one dimensionless coupling, α_s. Suppose a physical quantity, P, is calculated to n^{th} order in perturbation theory in two

renormalization schemes, then the nth order approximation is given by

$$P_n = \alpha_s(p_o + p_1\alpha_s + p_2\alpha_s^2 + \cdots + p_{n-1}\alpha_s^{n-1}) , \qquad (2.4)$$

in one scheme and by

$$P_n' = \alpha_s'(p_o' + p_1'\alpha_s' + p_2'\alpha_s'^2 + \cdots + p_{n-1}'\alpha_s'^{n-1}) , \qquad (2.5)$$

in the other scheme. The two couplings, defined in their respective renormalization schemes, can be related to each other

$$\alpha_s' = \alpha_s(1 + b_1\alpha_s + b_2\alpha_s^2 + \cdots + b_{n-1}\alpha_s^{n-1}) . \qquad (2.6)$$

Substituting Eq. (2.6) into Eq. (2.5) yields

$$\begin{aligned} P_n' = &\alpha_s(p_o'' + p_1''\alpha_s + p_2''\alpha_s^2 + \cdots + p_{n-1}''\alpha_s^{n-1}) \\ &+ \alpha_s(p_n''\alpha_s^n + \cdots + p_{2n-1}''\alpha_s^{2n-1}) . \end{aligned} \qquad (2.7)$$

As this is an approximation to the same quantity, P, the first n terms can be identified with P_n. Therefore the two approximations differ by terms of higher order

$$P_n' - P_n = \alpha_s(p_n''\alpha_s^n + \cdots + p_{2n-1}''\alpha_s^{2n-1}) . \qquad (2.8)$$

In QCD where α_s is large, this difference may be large and thereby lead to renormalization scheme dependence problems. In the Standard Model as in QED where the couplings are small, this is not so great a problem. In QCD where the strong coupling α_s is large, there will be renormalization scheme dependence problems. In the electroweak model as in QED where the couplings are small, this is not so great a problem.

CHAPTER 3
THE STANDARD MODEL

3.1 $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, and $\alpha_3(M_Z)$

The determination of the $SU(2)_L \times U(1)_Y$ couplings proceeds from the
Standard Model relations

$$\alpha_1(\mu) \equiv \frac{g_1^2(\mu)}{4\pi} = C^2 \frac{\alpha(\mu)}{\cos^2 \theta_W(\mu)} ,$$

$$\alpha_2(\mu) \equiv \frac{g_2^2(\mu)}{4\pi} = \frac{\alpha(\mu)}{\sin^2 \theta_W(\mu)} ,$$

(3.1.1)

where $\alpha(\mu) = e^2(\mu)/4\pi$ and C^2 is a normalization constant which equals 1
for the Standard Model and equals $\frac{5}{3}$ when the Standard Model is incorpo-
rated in grand unified theories of the $SU(N)$ and $SO(N)$ type [5]. What
is required to specify these couplings are the values of $\alpha(\mu)$ and $\sin^2 \theta_W(\mu)$
in the renormalization scheme employed (*i.e.*, $\overline{\text{MS}}$). The electromagnetic fine
structure constant ($\alpha_{em}^{-1} \approx 137.036$) is extrapolated from zero momentum scale
to a scale μ equal to M_Z in the present case. In pure QED with one species
of fermion with mass m the $\overline{\text{MS}}$ renormalized vacuum polarization function is
given by

$$\Pi(q^2) = \frac{\alpha(\mu)}{3\pi} (\ln\frac{\mu^2}{m^2} - 6 \int_0^1 dx \; x(1-x)\ln[1 - x(1-x)\frac{q^2}{m^2}]) .$$

(3.1.2)

The renormalized coupling $\alpha(\mu)$ is related to the fine structure constant α_{em}
as follows

$$\alpha_{em} = \frac{\alpha(\mu)}{1 + \Pi(0)} .$$

(3.1.3)

In the Standard Model where there are many species of charged fermions and charged gauge bosons, Eq. (3.1.3) generalizes to [14]

$$\alpha^{-1}(\mu) = \alpha_{em}^{-1} - \frac{2}{3\pi}\sum_f Q_f^2 \theta(\mu - m_f)\ln\frac{\mu}{m_f} + \frac{1}{6\pi} \ . \tag{3.1.4}$$

The effects of the strong interaction, which enter as a hadronic contribution to the vacuum polarization function, must be included also. The nonperturbative nature of the strong interaction at low momentum is handled by rewriting the hadronic contribution to the vacuum polarization at zero momentum as

$$\Pi^h(0) = (\Pi^h(0) - \Pi^h(q^2)) + \Pi^h(q^2) \ . \tag{3.1.5}$$

If q^2 is chosen large enough, $\Pi^h(q^2)$ can be calculated perturbatively. The terms $(\Pi^h(0) - \Pi^h(q^2))$ can then be related to the total cross section for $e^+e^- \to$ hadrons [14]. Using the optical theorem, one may write

$$\text{Im}\{\Pi^h(s)\} = \frac{s}{4\pi\alpha_{em}}\sigma(e^+e^- \to \text{hadrons}) \ , \tag{3.1.6}$$

where s is the square of the center of mass energy. For the process $e^+e^- \to \mu^+\mu^-$, the cross section is calculated to be (taking $m_\mu = 0$)

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha_{em}^2}{3s} \ . \tag{3.1.7}$$

In terms of the ratio of these two cross sections,

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \ , \tag{3.1.8}$$

one may write Eq. (3.1.6)

$$\text{Im}\{\Pi^h(s)\} = \frac{\alpha_{em}}{3}R(s) \ . \tag{3.1.9}$$

Using an unsubtracted dispersion relation for $\Pi^h(q^2)$ the combination $(\Pi^h(0) - \Pi^h(q^2))$ can be expressed as

$$\Pi^h(0) - \Pi^h(q^2) = \frac{q^2 \alpha_{em}}{3\pi} \int\limits_{4m_\pi^2}^{\infty} ds \, \frac{R(s)}{s(q^2 - s)} \ . \tag{3.1.10}$$

This can be evaluated using experimentally known data. This procedure yields a value

$$\alpha^{-1}(M_Z) = 127.9 \pm 0.3 \ . \tag{3.1.11}$$

The process independent, renormalized weak mixing angle $\sin^2 \theta_W$ of the on-shell scheme is defined to be

$$\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} \ , \tag{3.1.12}$$

where M_W and M_Z are the physical masses of the W and Z gauge bosons. Knowing the precise values of the W and Z boson masses and using the equation above provide one way of extracting the value of $\sin^2 \theta_W$. Alternatively, the bare relation involving the low energy Fermi constant measured in muon decay and the W boson mass

$$\frac{G_{\mu_o}}{\sqrt{2}} = \frac{e_o^2}{8 \sin^2 \theta_{W_o} M_{W_o}^2} \ , \tag{3.1.13}$$

may be corrected to order α and rewritten [15,16]

$$M_W = M_Z \cos \theta_W = (\frac{\pi \alpha_{em}}{\sqrt{2} G_\mu})^{\frac{1}{2}} \frac{1}{\sin \theta_W (1 - \Delta r)^{\frac{1}{2}}} \ , \tag{3.1.14}$$

with $(\pi \alpha_{em}/\sqrt{2} G_\mu)^{\frac{1}{2}} = 37.281$ GeV and Δr is a parameter containing order α radiative corrections and which depends on the mass of the top and Higgs. The radiative corrections represented by Δr can be viewed as accounting for the mismatch in the scales associated with the parameters of the relation. G_μ and α_{em} are low energy parameters whereas M_W and $\sin^2 \theta_W$ are associated

with the electroweak scale. One can absorb the radiative effects using the renormalization group by replacing G_μ and α_{em} with corresponding running parameters at M_Z

$$\frac{\pi\alpha(M_Z)}{\sqrt{2}G_\mu(M_Z)M_W^2\sin^2\theta_W} \approx 1 \ . \tag{3.1.15}$$

Combining Eqs. (3.1.14) and (3.1.15) gives

$$\Delta r \approx 1 - \frac{\alpha_{em}}{\alpha(M_Z)}\frac{G_\mu(M_Z)}{G_\mu} \ . \tag{3.1.16}$$

Using Eq. (3.1.4) and the fact that $G_\mu(M_Z) \approx G_\mu$ (see Section 2.8) gives an estimate of the size of the radiative corrections

$$\Delta r \approx 0.07 \ . \tag{3.1.17}$$

For large values of M_t and M_H ($M_t, M_H \gg M_Z$) [15,17]

$$\Delta r \approx 1 - \frac{\alpha_{em}}{\alpha(M_Z)} - \frac{3\alpha_{em}}{16\pi\sin^4\theta_W}\frac{M_t^2}{M_Z^2} + \frac{11\alpha_{em}}{48\pi\sin^2\theta_W}\ln\frac{M_H^2}{M_Z^2} \ . \tag{3.1.18}$$

A third way of extracting $\sin^2\theta_W$ is from neutral current experiments, among which deep inelastic neutrino scattering appears to provide the best determination. A running $\sin^2\theta_W(\mu)$ may be defined in $\overline{\text{MS}}$ and differs from the above $\sin^2\theta_W$ by order α corrections. The $\overline{\text{MS}}$ running W boson mass $m_W(\mu)$ and the corresponding physical mass M_W, identified as the simple pole at $q^2 = M_W^2$ of the W propagator, are related as follows

$$M_W^2 = m_W^2(\mu) + A_{WW}^T(M_W^2, \mu) \ , \tag{3.1.19}$$

where A_{WW}^T is the transverse part of the W self-energy. A similar relation holds for the Z boson. In $\overline{\text{MS}}$ renormalization, the following relation defines the running $\sin^2\theta_W(\mu)$

$$\sin^2\theta_W(\mu) = 1 - \frac{m_W^2(\mu)}{m_Z^2(\mu)} \ . \tag{3.1.20}$$

Equation (physmw) and its Z analog may be combined with Eq.(3.1.20) to give

$$\frac{\sin^2 \theta_W(\mu)}{\sin^2 \theta_W} = 1 - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left(\frac{A_{ZZ}^T(M_Z^2, \mu)}{M_Z^2} - \frac{A_{WW}^T(M_W^2, \mu)}{M_W^2} \right) . \tag{3.1.21}$$

An explicit expression relating $\sin^2 \theta_W$ and $\sin^2 \theta_W(M_W)$ is given in Ref. 18.

Another relation for $\sin^2 \theta_W(\mu)$ may be arrived at directly linking it to M_Z [19] or M_W [20]. In particular, if one chooses M_W as the input mass, then one introduces a radiative correction parameter $\Delta \hat{r}_W$ such that

$$\sin^2 \theta_W(M_Z)(1 - \Delta \hat{r}_W) = \sin^2 \theta_W(1 - \Delta r) , \tag{3.1.22}$$

from which it follows that

$$\sin^2 \theta_W(M_Z) = \frac{(37.271)^2}{M_W^2(1 - \Delta \hat{r}_W)} . \tag{3.1.23}$$

Similarly one can introduce a radiative correction $\Delta \hat{r}_Z$ if one chooses M_Z as the input mass

$$\sin^2 \theta_W(M_Z) \cos^2 \theta_W(M_Z)(1 - \Delta \hat{r}_Z) = \sin^2 \theta_W \cos^2 \theta_W(1 - \Delta r) . \tag{3.1.24}$$

A fit to all neutral current data gives

$$\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.0011 , \tag{3.1.25}$$

for arbitrary M_t [21]. Using these values of $\alpha(M_Z)$ and $\sin^2 \theta_W(M_Z)$ yields

$$\alpha_1(M_Z) = 0.01698 \pm 0.00009 ,$$
$$\tag{3.1.26}$$
$$\alpha_2(M_Z) = 0.03364 \pm 0.0002 .$$

The value of the strong coupling has not been determined experimentally as well as α_1 and α_2. The nonperturbative nature of low energy QCD leads to relatively large uncertainties in its determination. Different processes for the extraction of α_3 yield significantly different results. Unless otherwise stated, the

value of α_3 used in the runs will be that obtained from a Gaussian weighted average of the results of several processes, including e^+e^- scattering into hadrons [22], heavy quarkonium decay [23], deep inelastic scattering [24], and e^+e^- scattering into jets [25]. The value found is [6]

$$\alpha_3(M_Z) = 0.113 \pm 0.004 . \tag{3.1.27}$$

3.2 Yukawas

To take full account of the Yukawa sector in running all the couplings, initial values for the Yukawa couplings are necessary. They must be extracted from physical data such as quark masses and CKM mixing angles. Furthermore, the interesting parameters to be plotted must be determined step by step in the process of running to Planck mass. These two procedures are not unrelated and require the diagonalization of the up-type, down-type, and leptonic Yukawa matrices.

Machacek and Vaughn's [7] convention are used where the interaction Lagrangian for the Yukawa sector is

$$\mathcal{L} = \bar{u} \mathbf{Y}_u \tilde{\Phi}^\dagger Q + \bar{d} \mathbf{Y}_d \Phi^\dagger Q + \bar{e} \mathbf{Y}_e \Phi^\dagger L + \text{h.c.} . \tag{3.2.1}$$

The Yukawa couplings are given in terms of 3×3 complex matrices. After electroweak symmetry breaking, these translate into the quark and lepton masses

$$\mathbf{Y}_e = \frac{\sqrt{2}}{v} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} ,$$

$$\mathbf{Y}_d = \frac{\sqrt{2}}{v} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} , \tag{3.2.2}$$

$$\mathbf{Y}_u = \frac{\sqrt{2}}{v} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} V ,$$

where V is the CKM matrix which appears in the charged current

$$j_\mu^+ \sim \bar{u}_L \gamma_\mu V d_L \ . \tag{3.2.3}$$

It is a unitary 3×3 matrix often parametrized as follows

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \ , \tag{3.2.4}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$, $i = 1, 2, 3$.

The entries of the parametrized CKM matrix can be related simply to the experimentally known CKM entries. The particle data book [21] gives the following ranges of values (assuming unitarity) for the magnitudes of the elements of the CKM matrix

$$|V| = \begin{pmatrix} 0.9747 - 0.9759 & 0.218 - 0.224 & 0.001 - 0.007 \\ 0.218 - 0.224 & 0.9734 - 0.9752 & 0.030 - 0.058 \\ 0.003 - 0.019 & 0.029 - 0.058 & 0.9983 - 0.9996 \end{pmatrix} \ . \tag{3.2.5}$$

These ranges of values can be converted to bounds for s_i, $i = 1, 2, 3$, and $\sin\delta$. These bounds are arrived at by finding values for the four angles such that the entries of the CKM matrix obtained from these satisfy the conditions imposed by Eq.(3.2.5). One finds

$$0.2188 \le \sin\theta_1 \le 0.2235 \ ,$$

$$0.0216 \le \sin\theta_2 \le 0.0543 \ , \tag{3.2.6}$$

$$0.0045 \le \sin\theta_3 \le 0.0290 \ .$$

However, the accuracy with which $|V|$ is known does not constrain $\sin\delta$. A set of angles $\{\theta_1, \theta_2, \theta_3, \delta\}$ was chosen that falls within the ranges quoted above. The initial data needed to run the Yukawa elements are extracted from the CKM matrix and the quark masses. A problem arises though for the mixing angles, which was solved for the quark masses (see Section 2.5), in that it is not clear at what scale the chosen initial values for these angles should

be considered known. However, it is observed that for the whole range of initial values, the running of the mixing angles is quite flat, with a perceptible increase in θ_2 between M_W and the Planck scale for higher top masses. This is in accordance with the angles being related to ratios of quark masses, and therefore, the exact knowledge of that scale (or lack thereof) is not as critical as might be feared *a priori*.

3.3 Quark and Lepton Masses

There are large theoretical uncertainties in the extraction of the masses of the three lightest quarks from experiment. They are determined by chiral perturbation techniques and QCD spectral sum rules [26,27]. In the following, the $\overline{\text{MS}}$ running masses at 1 GeV of the three lightest quark will be taken to be [6]

$$m_u(1 \text{ GeV}) = 5.2 \pm 0.5 \text{ MeV} ,$$

$$m_d(1 \text{ GeV}) = 9.2 \pm 0.5 \text{ MeV} , \qquad (3.3.1)$$

$$m_s(1 \text{ GeV}) = 194 \pm 4 \text{ MeV} ,$$

For the charm and bottom, the nonrelativistic bound state approximation may be applied. One speaks of physical masses and associates these with the pole of the quark propagator. A weighted average, based on results from J/ψ and Υ sum rules [28] and from heavy-light, B and B^*, D and D^* meson masses and semileptonic B and D decays [29], yields the following physical and running masses at 1 GeV for the charm and bottom [6]

$$M_c = 1.60 \pm 0.05 \text{ GeV}, \quad m_c(1 \text{ GeV}) = 1.41 \pm 0.06 \text{ GeV} ,$$

$$M_b = 4.89 \pm 0.04 \text{ GeV}, \quad m_b(1 \text{ GeV}) = 6.33 \pm 0.06 \text{ GeV} , \qquad (3.3.2)$$

Although the associated errors on these averages are restrictive, there is larger uncertainty in the actual central values. For example, in the bottom quark, values as small as $m_b(1 \text{ GeV}) = 5.7$ GeV and as large as $m_b(1 \text{ GeV}) = 6.5$

GeV are acceptable. In the case of the lighter quarks, their mass ratios are known more accurately than the actual value of their respective masses. The ratios $m_d/m_u = 1.8$ and $m_s/m_d = 21$ are widely accepted.

The physical (pole) masses of the leptons are very well known [21]

$$M_e = 0.51099906 \pm 0.00000015 \text{ MeV} \;,$$

$$M_\mu = 105.658387 \pm 0.000034 \text{ MeV} \;, \qquad (3.3.3)$$

$$M_\tau = 1.7841^{+0.0027}_{-0.0036} \text{ GeV} \;.$$

These masses represent an example of a physical quantity as discussed in Chapter 2. Indeed expressed in terms of the renormalized parameters (*i.e.*, masses and couplings) of the theory, the physical mass is just the simple pole of the relevant field's propagator. Suppose $S(\not{p}; g, m, \mu)$ is the renormalized propagator of some fermion with a simple pole at $\not{p} = M(g, m, \mu)$ such that

$$S(\not{p}; g, m, \mu) = \frac{z(\not{p}; g, m, \mu)}{\not{p} - M(g, m, \mu)} \;, \qquad (3.3.4)$$

with finite residue, $z(\not{p} = M; g, m, \mu)$. The relevant renormalization group equation is

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + m\gamma_m\frac{\partial}{\partial m} + 2\gamma)S(\not{p}; g, m, \mu) = 0 \;, \qquad (3.3.5)$$

where γ is the anomalous dimension and γ_m is the mass anomalous dimension. Inserting Eq. (3.3.4) in Eq.(3.3.5), then multiplying the resulting expression by $(\not{p} - M)^2$, and lastly setting $\not{p} = M$ gives the renormalization group equation for M

$$\mu\frac{d}{d\mu}M(g, m, \mu) = (\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + m\gamma_m\frac{\partial}{\partial m})M(g, m, \mu) = 0 \;. \qquad (3.3.6)$$

These values are used to determine initial data for the running masses. Some authors neglect QED corrections and use the physical values for the running

values at $\sim M_Z$, which introduces only a small error. By calculating the one-loop self-energy corrections, one arrives at a QED relation between the running $\overline{\text{MS}}$ masses and the corresponding physical masses

$$m_l(\mu) = M_l[1 - \frac{3\alpha(\mu)}{4\pi}(\ln\frac{\mu^2}{m_l^2} + \frac{4}{3})] \ . \qquad (3.3.7)$$

Choosing $\mu = 1$ GeV as in the quark mass case and using Eqs. (3.3.7) and (3.1.4) yields the running lepton masses (taking $m_l = M_l$ in the log term above is an appropriate approximation to order α)

$$m_e(1 \text{ GeV}) = 0.4960 \text{ MeV} \ ,$$

$$m_\mu(1 \text{ GeV}) = 104.57 \text{ MeV} \ , \qquad (3.3.8)$$

$$m_\tau(1 \text{ GeV}) = 1.7835 \text{ GeV} \ .$$

3.4 Top and Higgs Masses

The Higgs boson and top quark masses have not been measured directly at present; however, their values affect radiative corrections such as Δr. Consistency with experimental data on $\sin^2\theta_W$ requires $M_t < 197$ GeV for $M_H = 1$ TeV at 99% CL assuming no physics beyond the Standard Model [30]. Precision measurements of the Z mass and its decay properties combined with low energy neutral current data have been used to set stringent bounds on the top quark mass within the minimal Standard Model. A global analysis of this data yields $M_t = 122^{+41}_{-32}$ GeV, for all allowed values of M_H [31]. Recent direct search results set the experimental lower bound $M_t \gtrsim 91$ GeV. As for the Higgs, the analysis of Ref. 31 gives the restrictive bound, $M_H \lesssim 600$ GeV, if $M_t < 120$ GeV, and $M_H < 6$ TeV, for all allowed M_t. Since perturbation theory breaks down for $M_H \gtrsim 1$ TeV, the latter bound on the Higgs boson mass is not necessarily meaningful. LEP data set a lower bound on the Higgs boson mass of 48 GeV [32].

In the present analysis, initial values of the $\overline{\text{MS}}$ running top quark mass m_t and of the scalar quartic self-coupling λ at M_Z are chosen arbitrarily (consistent with the bounds quoted above). As noted earlier in Chapter 2, these running parameters are not equal to their physical counterparts. However, any reasonable prediction for the masses of the top quark and of the Higgs boson that may come from this analysis should be that of experimentally relevant, physical masses. Therefore, formulas similar to Eq. (3.3.7) relating $\overline{\text{MS}}$ running parameters to physical masses are needed. To calculate the physical or pole mass of the top quark, the following equation is used [33]

$$\frac{M_t}{m_t(M_t)} = 1 + \frac{4}{3}\frac{\alpha_s(M_t)}{\pi} + [16.11 - 1.04\sum_{i=1}^{5}(1 - \frac{M_i}{M_t})](\frac{\alpha_s(M_t)}{\pi})^2 , \quad (3.4.1)$$

where M_i, $i = 1, \ldots, 5$, represent the masses of the five lighter quarks. Likewise the physical mass of the Higgs boson can be extracted from the following relation [34]

$$\lambda(\mu) = \frac{G_\mu}{\sqrt{2}}M_H^2(1 + \delta(\mu)) , \quad (3.4.2)$$

where $\delta(\mu)$ contains the radiative corrections. Its form is rather elaborate and it is relegated to Appendix B. Equations (3.4.1) and (3.4.2) are highly nonlinear functions of M_t and M_H, respectively. Their solution requires numerical routines that are described in Appendix C.

At the one loop level, the gauge couplings are unaffected by the other couplings in the theory. On the other hand, the Yukawa couplings are affected at one loop by both the gauge and Yukawa couplings. Since the top Yukawa coupling is at least as big as the gauge couplings at low energy, that means the running of the Yukawas is sensitive to mostly the top Yukawa and the QCD gauge couplings. Thus one expects the mass and mixing relations just described to be sensitive to the value of the top quark mass. The Higgs quartic

self-coupling enters in the running of the other couplings only at the two loop level, so that its effect on the quark and lepton parameters is small. However, its own running is very sensitive to the top quark mass; it can become negative as easily as it can blow up, corresponding to vacuum instability or to strong self-interaction of the Higgs (triviality bound), respectively [35]. The discovery of the Higgs with mass outside these bounds would be a signal for physics beyond the Standard Model. The graphs in Figs. 1-4 summarize these bounds for representative values of the top quark mass. For example, if $M_t = 150$ GeV, one can see from the corresponding plot that a Higgs mass between 95 and 150 GeV need not imply any new physics up to Planck scale. However, if the Higgs were observed outside of this range, then some new physics must appear at the scale indicated by the curve, either because of vacuum instability if $M_H < 95$ GeV or because the Higgs interaction becomes too strong if $M_H > 150$ GeV. It is amusing to note that it is for comparable values of the top and Higgs masses that these bounds are least restrictive, but it is important to emphasize that a high value of the top with a relatively low value of the Higgs necessarily indicates the presence of new physics within reach of the SSC.

3.5 Vacuum Expectation Value

The vacuum expectation value (vev) of the scalar field may be extracted from the well known lowest order relation

$$v = (\sqrt{2} G_\mu)^{-\frac{1}{2}} = 246.22 \text{ GeV} . \tag{3.5.1}$$

From the very well measured value of the muon lifetime, $\tau_\mu = 2.197035 \pm .000040 \times 10^{-6}$ s [21], the Fermi constant can be extracted using the following formula [36]

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f(\frac{m_e^2}{m_\mu^2})(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2})[1 + \frac{\alpha(m_\mu)}{2\pi}(\frac{25}{4} - \pi^2)] , \tag{3.5.2}$$

where

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2\ln x \ , \qquad (3.5.3)$$

giving

$$G_\mu = 1.16637 \pm 0.00002 \times 10^{-5} \ \text{GeV}^{-2} \ . \qquad (3.5.4)$$

This parameter may be viewed as the coefficient of the effective four-fermion operator for muon decay in an effective low energy theory

$$\frac{G_\mu}{\sqrt{2}}[\bar{\nu}_e\gamma^\beta(1-\gamma_5)e][\bar{\mu}\gamma_\beta(1-\gamma_5)\nu_\mu] \ . \qquad (3.5.5)$$

A direct calculation (e.g., in the Landau gauge) of the electromagnetic corrections yields that the operator is finitely renormalized (i.e., G_μ does not run) [16,37]. Another way to see this is by using a Fierz transformation to rewrite the above expression

$$\frac{G_\mu}{\sqrt{2}}[\bar{\nu}_e\gamma^\beta(1-\gamma_5)\nu_\mu][\bar{\mu}\gamma_\beta(1-\gamma_5)e] \ . \qquad (3.5.6)$$

The neutrino current does not couple to the photon field, and the $e - \mu$ current is conserved and is hence not multiplicatively renormalized.

An initial value is needed for the running vacuum expectation value at some scale μ. Wheater and Llewellyn Smith [38] consider muon decay to order α in the context of the full electroweak theory and derive an equation relating an $\overline{\text{MS}}$ running G_μ to the experimentally measured value. From this formula one can extract a value for $v(M_Z)$. However, the formula is derived in the 't Hooft-Feynman gauge, and the evolution equation, Eq. (I.18) of Appendix A for the vev, is valid only in the Landau gauge (see Appendix D). Nevertheless, motivated by the discussion of the previous paragraph, the initial condition for the vev is chosen to be $v(M_W) = 246.22$ GeV. Using the initialization algorithm (see Appendix C), one arrives at $v(M_Z)$. It is found that this procedure leads

to no significant correction, and one therefore takes, *ab initio*, $v(M_Z) = 246.22$ GeV.

3.6 Thresholds

In mass independent renormalization schemes, the running couplings are unphysical. From the decoupling theorem [39] one expects the physics at energies below a given mass scale to be independent of the particles with masses higher than this threshold. Therefore, for a correct interpretation of these running couplings, one must take into account the thresholds [40,41,42]. For the electroweak threshold, one loop matching functions [42] are used with the two loop beta functions valid in the Standard Model regime below the SUSY scale. These matching functions are obtained in $\overline{\text{MS}}$ renormalization by integrating out the heavy gauge fields in such a way that the remaining effective action is invariant under the residual gauge group [41]. At the electroweak threshold, near M_W, the heavy gauge fields and the top quark are integrated out. Below this threshold there is an effective $SU(3)_C \times U(1)_{EM}$ theory. Thresholds in this region are obtained by integrating out each quark to one loop at a scale equal to its physical mass. At these scales the one loop matching functions in the gauge couplings vanish and the threshold dependence appears through steps in the number of quark flavors [43] as the renormalization group scale passes each physical quark mass.

In the SM runs (see next section), in which an analysis of the relative importance of including a proper treatment of thresholds effects instead of using a simple step function technique, it is found that the former method does not improve significantly over the latter [6].

3.7 Analysis and Results

The results of numerically integrating the β functions of the Standard Model parameters from 1 GeV to Planck mass are depicted in the following figures. For most of these plots, the arbitrary choice, $M_t = M_H = 100$ GeV, is made. Figure 5 displays the evolution of the inverse of each of the three gauge couplings including the associated uncertainties in their values. In it, one sees the "GUT triangle" signifying the absence of grand unification, assuming the Standard Model as an effective theory in the desert up to the Planck scale. Here, the differences between one- and two-loop evolution appear in the high energy regime. Differences are also present for the strong coupling at low energies where it becomes large. Note that the uncertainties do not fill in the "GUT triangle." Figures 6, 7, and 8 display the evolution of the light mass fermions (m_e, m_u, and m_d), the intermediate mass fermions (m_μ and m_s), and the heavy mass fermions (m_τ, m_c, and m_b), respectively. The largest differences between one-loop and two-loop evolution occur in the bottom, charm, and strange quark masses in these cases. In Fig. 9, the quartic self-coupling λ and the top Yukawa coupling y_t for ($M_t = 100$ GeV, $M_H = 100$ GeV) and for ($M_t = 200$ GeV, $M_H = 195$ GeV) are plotted. These two couplings are the only unknown parameters of the Standard Model. The effects of changing the values of M_t and M_H in the analyses of the running of the other parameters have been studied. It is observed that, for any M_t between 100 GeV and 200 GeV, varying M_H, while maintaining perturbativity and vacuum stability, did not affect appreciably the evolution of any of the other parameters. However, changing M_t itself showed a significant difference in the running of the heavier quarks. In particular, Fig. 10 shows that the intersection point between the bottom quark and the τ lepton moves down to a lower scale for higher

top quark masses. This is expected since from Eq. (I.9) one can see that the bottom type Yukawas are driven down by an increased top Yukawa. This is to be contrasted with the SUSY GUT case in which the bottom Yukawa β function is such that this crossing point is shifted toward a higher scale with an increased top mass. The relation $m_b = m_\tau$ (I) is the most natural one in the $SU(5)$ theory [44], and it could be expected to be valid at scales where the Standard Model gauge couplings are the closest to one another. Its validity is examined for three different physical values of the top and Higgs masses in the Standard Model. The noteworthy feature of Fig. 10 is that this simplest of the $SU(5)$ relations is valid at an energy scale many orders of magnitude removed from that at which the gauge couplings tend to converge. This result is vastly different from that of the original investigations [45]. This work improves upon that work by including two loop effects in the running of the quark Yukawas, by taking into account the full Yukawa sector, and most importantly by incorporating QCD corrections in the extraction of the bottom quark mass. Other mass relations were studied in the SM context [46]: (II) $m_d = 3m_e$, $3m_s = m_\mu$ [47], (III) $\tan\theta_{Cabbibo} = (m_d/m_s)^{1/2}$ [48], and (IV) $V_{cb} = (m_c/m_t)^{1/2}$ [49]. There is no scale at which all of these can be satisfied. The scale at which relation (I) tends to be satisfied does not coincide with that at which the others are valid. Still the disagreement is never too large. In an $SU(5)$ SUSY GUT model, the equality of the bottom and τ Yukawas at the scale of unification will be used to get bounds on the top and Higgs masses [3]. Lastly, the running of the CKM angles is displayed in Fig. 11. The initial data used are $\sin\theta_1 = 0.2206$, $\sin\theta_2 = 0.0298$, and $\sin\theta_3 = 0.0106$. Also δ has been taken to be 90°. which corresponds to the case of maximal CP violation. As

mentioned in Section 2.4, the evolution curves for these angles are effectively flat.

In the present case of the Standard Model, it is found that two-loop running of the parameters does at times improve on the one-loop running. Indeed, the differences of several parameters in their one- versus two-loop values at various scales have been tabulated, for the cases ($M_t = 100$ GeV, $M_H = 100$ GeV) and ($M_t = 200$ GeV, $M_H = 195$ GeV). Table 1 illustrates the difference between one-loop and two-loop running in the ratio m_b/m_τ, for the three scales 10^2 GeV, 10^4 GeV, and 10^{16} GeV. Clearly, the difference between one- and two-loop results is more pronounced at higher scales, as expected. Over all these scales the difference is never less than 5.5%. Note that the ratio becomes equal to one well below the scale of grand unification as noted above in the discussion of Fig. 10. Table 2 presents a similar comparison for the top Yukawa. Here, two loops represent a smaller correction with the difference at all scales always being less than 5%. Finally, Table 3 displays the same analysis for α_s for the case $M_t = M_H = 100$ GeV. No appreciable deviation from the tabulated values is observed for any $M_t \lesssim 200$ GeV (except in the low energy regime where the difference is at most $\sim 4\%$). At scales $\lesssim M_Z$, the inclusion of two loops is important in the evolution of the strong coupling (and of the quark masses). Indeed, it is found that the pure QCD three-loop contribution is also significant and therefore include it in the running of the strong coupling and of the quark masses in the low energy region. As seen in this table, the combined two and three loops in the low energy regime account for a 17% difference at 1 GeV in α_s.

Although in the cases considered in these last two tables there does not appear to be a significant difference in two-loop over one-loop evolution at

scales above M_Z, the first table does show a 10% difference at the scale 10^{16} GeV. The effects of using a naive step approximation *vs.* a proper treatment of thresholds are numerically unimportant for the cases discussed above. Indeed they are less important than the two-loop effects.

Table 1: m_b/m_τ						
	$M_t = 100$ GeV			$M_t = 200$ GeV		
	10^2 GeV	10^4 GeV	10^{16} GeV	10^2 GeV	10^4 GeV	10^{16} GeV
One loop	1.879	1.455	0.8081	1.868	1.392	0.6647
Two loop	1.782	1.348	0.7336	1.769	1.285	0.6047

Table 2: y_t						
	$M_t = 100$ GeV			$M_t = 200$ GeV		
	10^2 GeV	10^4 GeV	10^{16} GeV	10^2 GeV	10^4 GeV	10^{16} GeV
One loop	0.5405	0.4160	0.1928	1.133	0.9780	0.7145
Two loop	0.5405	0.4071	0.1842	1.143	0.9700	0.6816

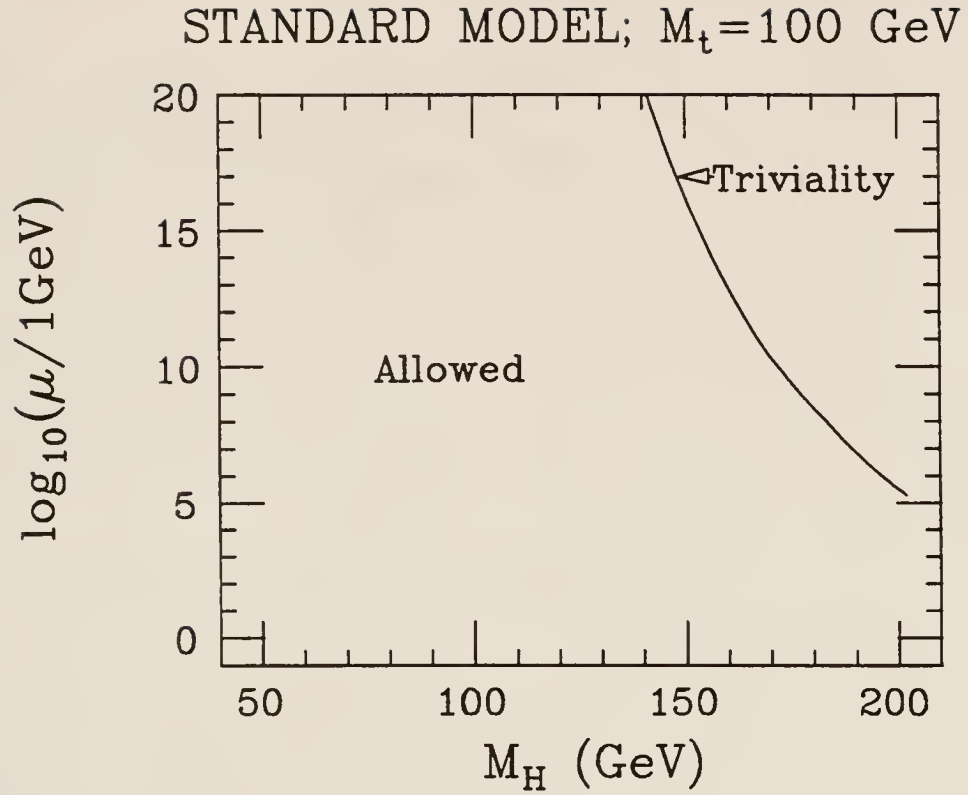| Table 3: α_s				
	1 GeV	10^2 GeV	10^4 GeV	10^{16} GeV
One loop	0.3128	0.1118	0.07103	0.02229
Two and three loop	0.3788	0.1117	0.07039	0.02208

Figure 1. Vacuum stability and triviality bounds on the Higgs mass for $M_t = 100$ GeV giving scales of expected new physics beyond the Standard Model.

Figure 2. Vacuum stability and triviality bounds on the Higgs mass for $M_t = 125$ GeV giving scales of expected new physics beyond the Standard Model.

Figure 3. Vacuum stability and triviality bounds on the Higgs mass for $M_t = 150$ GeV giving scales of expected new physics beyond the Standard Model.
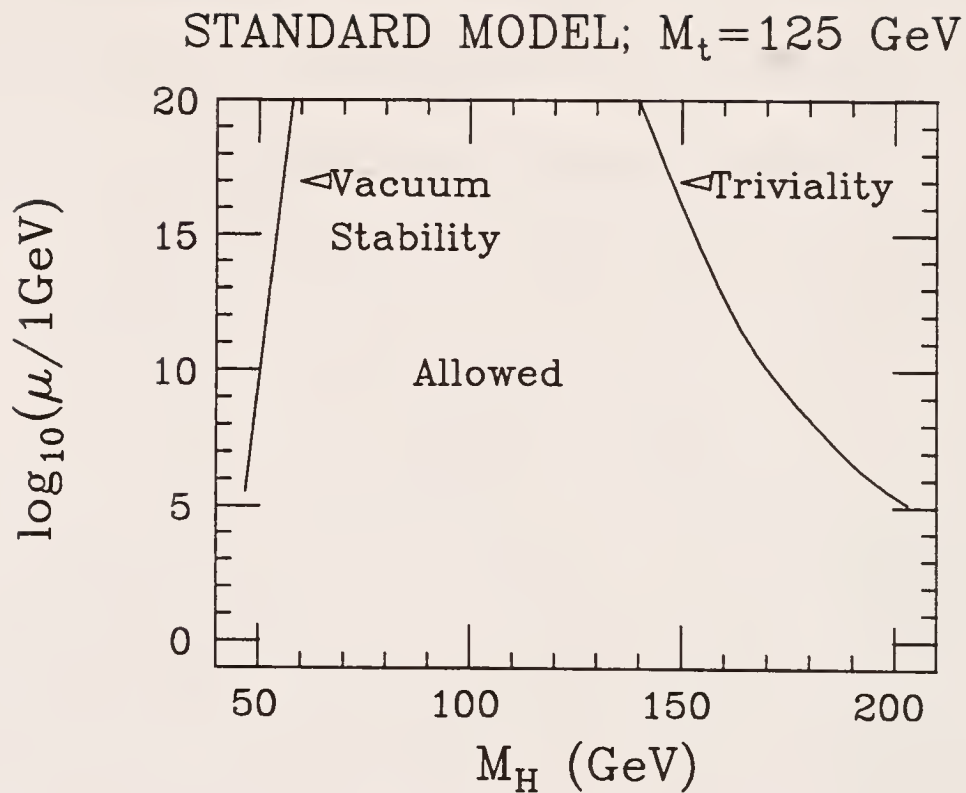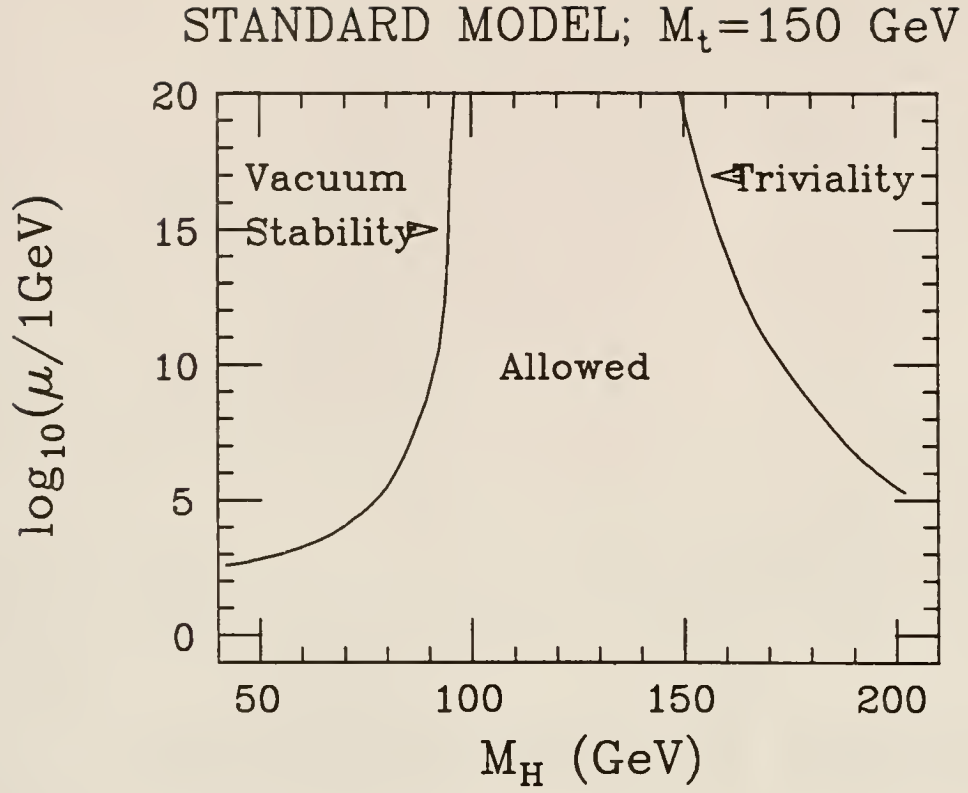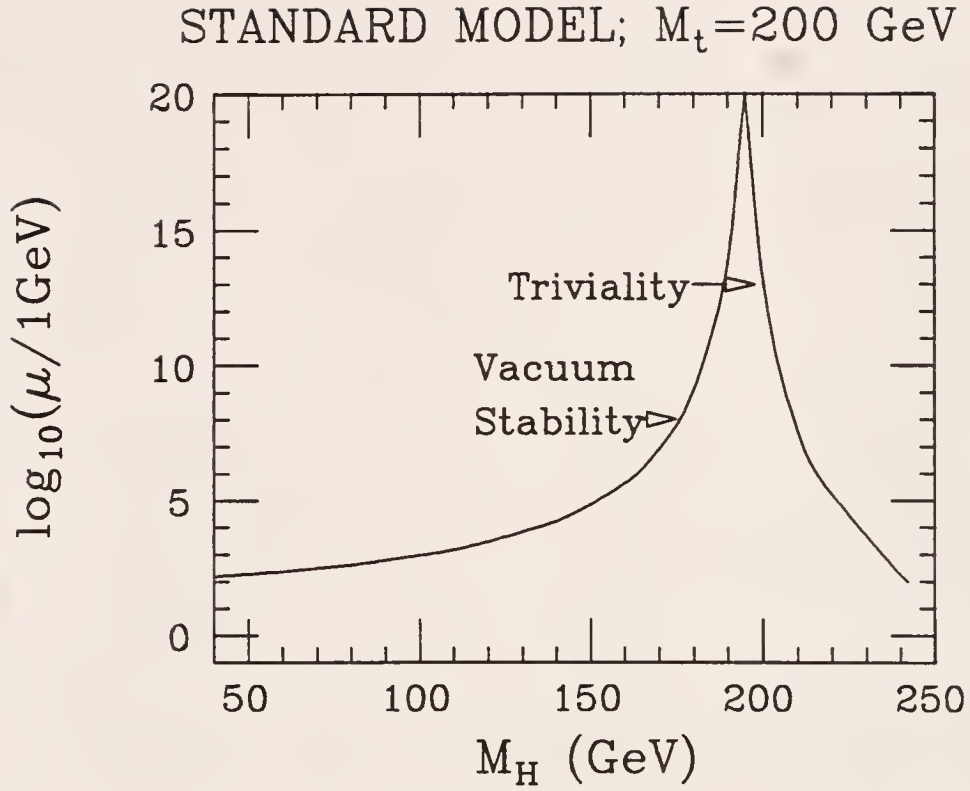
Figure 4. Vacuum stability and triviality bounds on the Higgs mass for $M_t = 200$ GeV giving scales of expected new physics beyond the Standard Model.
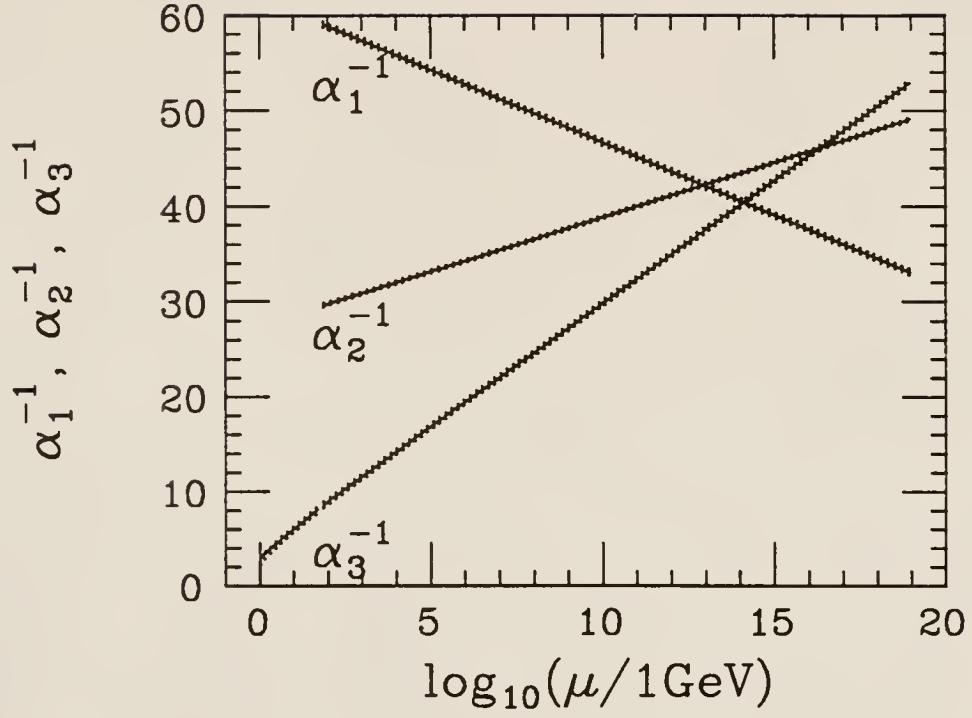
Figure 5. Running of the inverse gauge couplings using their propagated experimental errors for the two-loop case only.
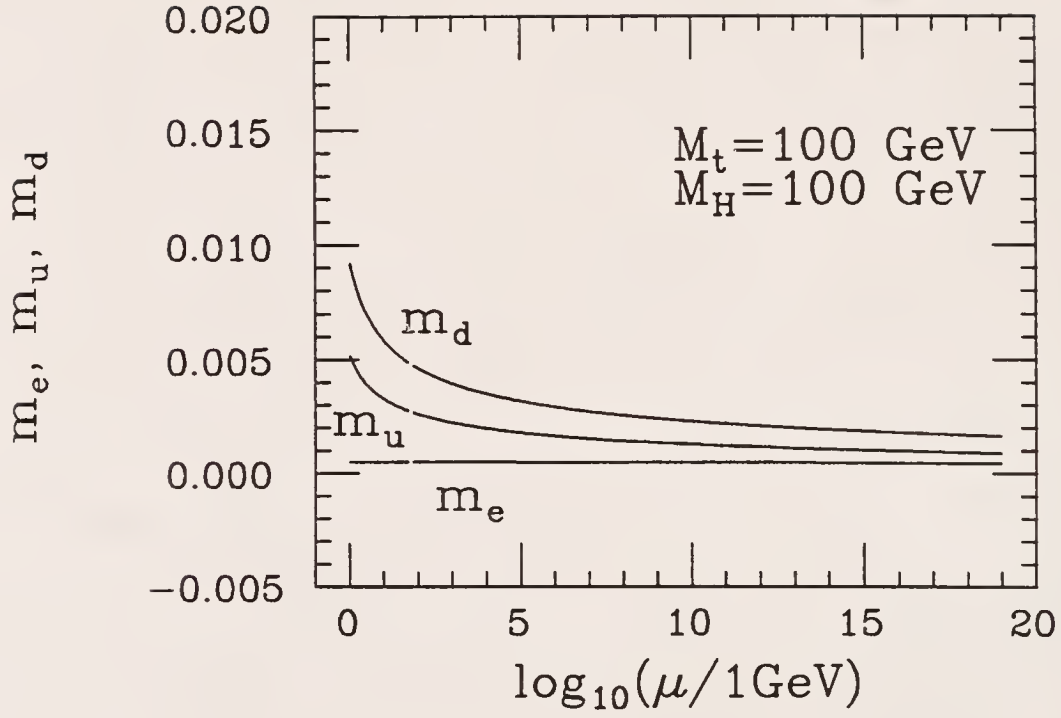
Figure 6. Light quark and lepton masses for $M_t = 100$ GeV and $M_H = 100$ GeV.

Figure 7. Intermediate quark and lepton masses for $M_t = 100$ GeV and $M_H = 100$ GeV.

Figure 8. Heavy quark and lepton masses for $M_t = 100$ GeV and $M_H = 100$ GeV.

Figure 9. Top Yukawa and scalar quartic couplings.

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Figure 10. Plot of m_b/m_τ as a function of scale in the Standard Model for various top and Higgs masses.

Figure 11. CKM mixing angles for $M_t = 100$ GeV and $M_H = 100$ GeV.

CHAPTER 4
THE MSSM

4.1 The Supersymmetric Standard Model

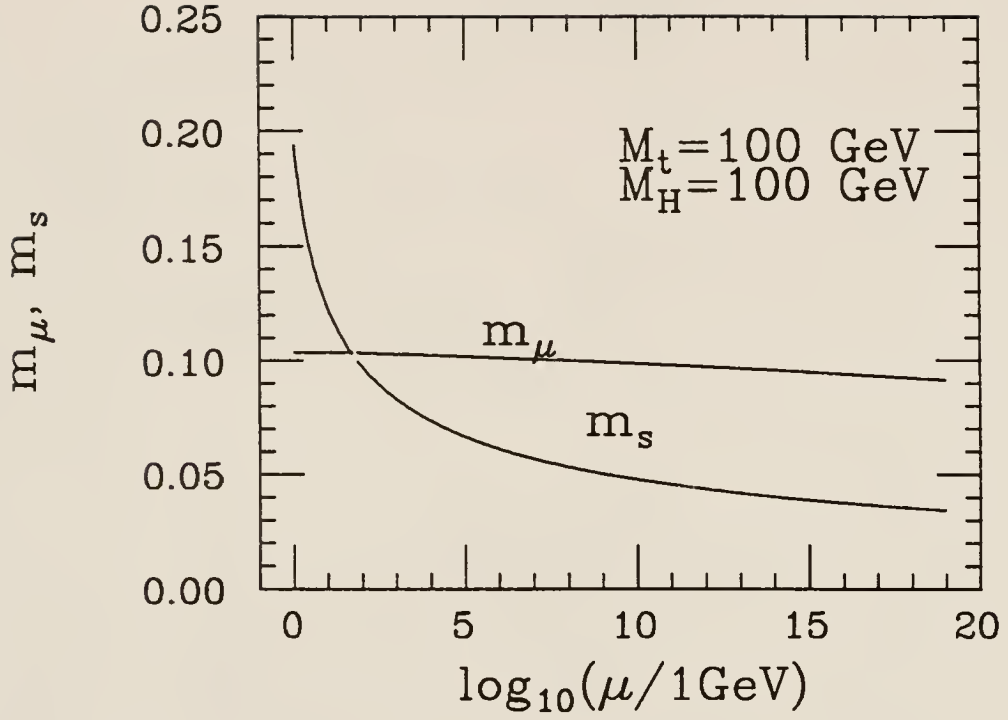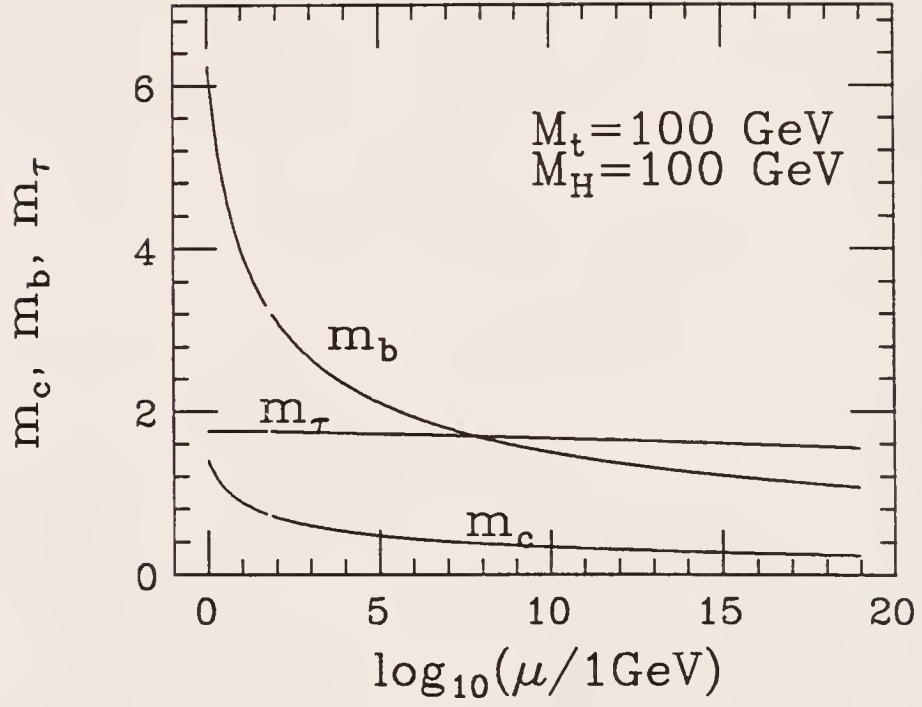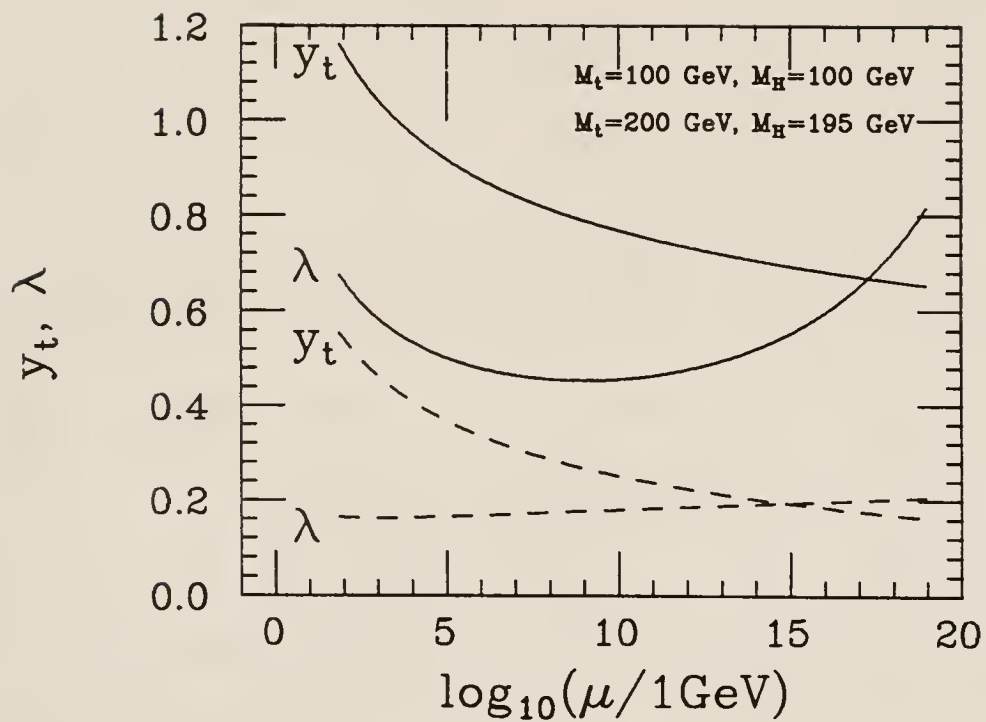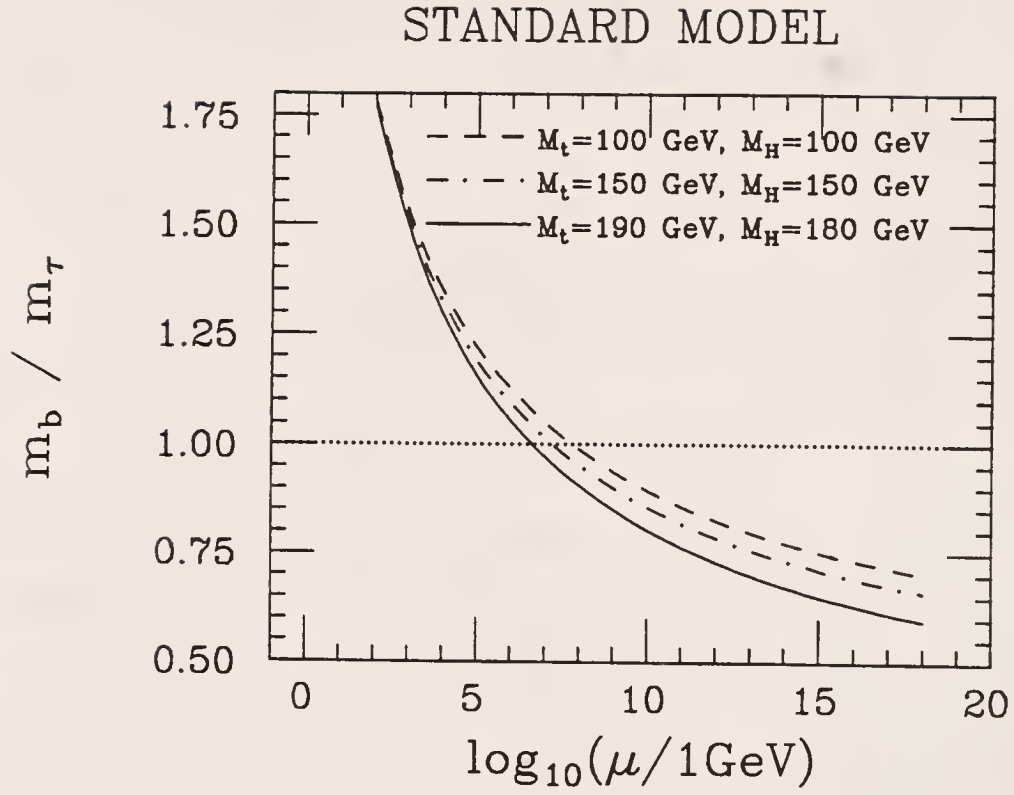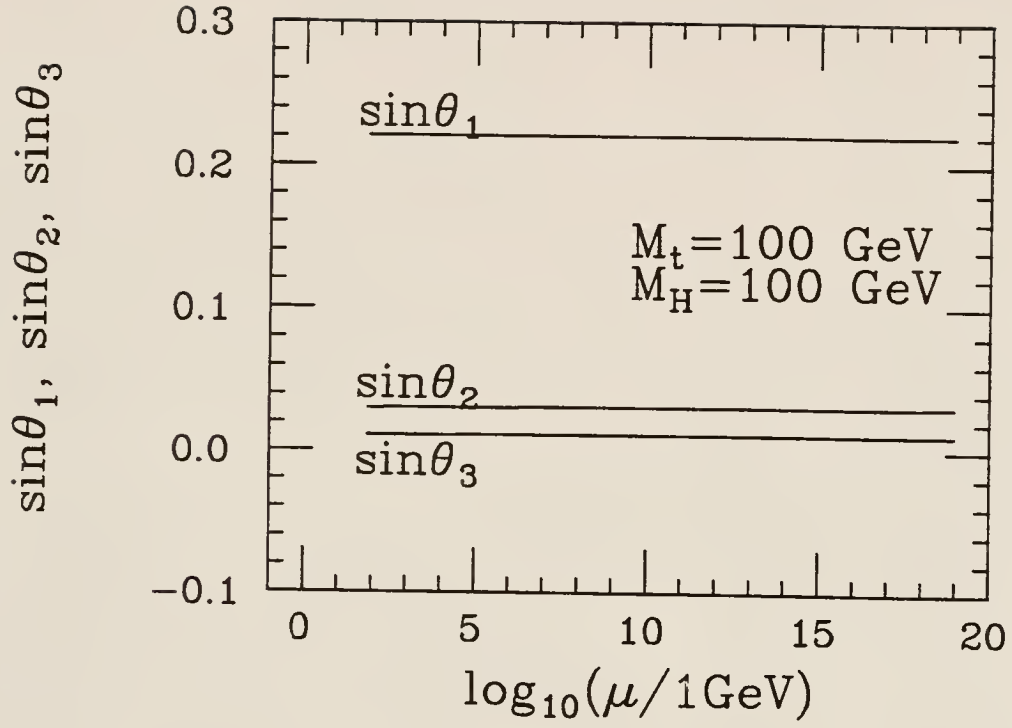In the minimal supersymmetric extension of the standard model (MSSM), every particle has a supersymmetric partner, their spins differing by a half [50]. Also required is a second Higgs field with opposite hypercharge to the first as the superpotential cannot contain both a field and its complex conjugate. The second Higgs is also needed for anomaly cancellation and to give this sector a mass. For renormalizable theories, the superpotential can have at most degree three interactions. The superpotential for the MSSM is (suppressing the $SU(2)$ and Weyl metrics)

$$W = \hat{\bar{u}} \mathbf{Y}_u \hat{\Phi}_u \hat{Q} + \hat{\bar{d}} \mathbf{Y}_d \hat{\Phi}_d \hat{Q} + \hat{\bar{e}} \mathbf{Y}_e \hat{\Phi}_d \hat{L} + \breve{\mu} \hat{\Phi}_u \hat{\Phi}_d + h.c. \ , \qquad (4.1.1)$$

where the hat indicates a chiral superfield and the overline denotes a left-handed CP conjugate of a right-handed field, $\overline{\psi} = i\sigma_2 \psi_R^*$. The usual Yukawa interactions are accompanied by new Yukawa interactions among the scalar quarks and leptons and the Higgsinos in the supersymmetric Lagrangian. There are also new gauge Yukawa interactions involving the gauginos. The new purely scalar interactions form the scalar potential which is positive definite in supersymmetric theories. The scalar potential will be discussed in a subsequent section.

A remarkable aspect of supersymmetry is that all these new interactions require no new couplings. The $\breve{\mu}$ term is only present to avoid a Peccei-Quinn

(PQ) symmetry. Omitting it would lead to exact PQ symmetry and to a visible axion which is experimentally ruled out. An alternative way to break the PQ symmetry is to omit the $\check{\mu}$ term and add an explicit soft symmetry breaking term, $m_3^2 \Phi_u \Phi_d$. The $\check{\mu}$ can be interpreted dynamically as essentially the vacuum expectation value of a singlet chiral superfield, $\hat{N}$, through the following additional interactions

$$\lambda \hat{N} \hat{\Phi}_u \hat{\Phi}_d + \kappa \hat{N} \hat{N} \hat{N} . \tag{4.1.2}$$

This approach also provides a natural explanation for $\check{\mu} \sim \mathcal{O}(M_W)$. The cubic term now explicitly breaks the PQ symmetry. Table 4 displays the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the chiral (all left-handed) and vector superfields of the MSSM.

4.2 Procedure

Bounds are presented for the mass of the top quark in a minimal supersymmetric extension of the Standard Model (MSSM) with minimal Higgs structure in the context of a grand unified theory (GUT) by numerically evolving the couplings using their renormalization group equations. This analysis improves on previous endeavors by taking full account of the Yukawa sector.

In the expectation that the Standard Model is only the low energy manifestation of some yet unknown GUT or of a possible supersymmetric (SUSY) extension thereof, the three couplings g_3, g_2, and g_1 corresponding to the Standard Model gauge groups, $SU(3)_C \times SU(2)_L \times U(1)_Y$, should meet at some large grand unification scale. Using the accepted values and associated errors of these couplings unification is observed in the SUSY-GUT case but not in the pure GUT case, as noted by several groups [1,51] (see Fig. 12). However, this should not be viewed as proof of supersymmetry since given the values of

α_1, α_2, α_3 at some scale, and three unknowns (the value of α at the unification scale, the unification scale, and an extra scale such as the SUSY scale) there is always a solution. The exciting aspect of the analysis of Ref. 1 is the numerical output, namely a low SUSY scale, M_{SUSY}, and a perturbative solution below the Planck scale which does not violate proton decay bounds [52].

Furthermore, in the context of a minimal GUT [44] there are constraints on the Yukawa couplings at the scale of unification. One first restrict oneself to an SU(5) SUSY-GUT [53] where y_b and y_τ, the bottom and τ Yukawa couplings, are equal at unification. The crossing of these renormalization group flow lines is sensitive to the physical top quark mass, M_t. This can be seen in the down-type Yukawa renormalization group equation (above M_{SUSY}, for example), from which the evolution of y_b is extracted, since the top contribution is large and appears already at one loop through the up-type Yukawa dependence

$$
\begin{aligned}
\frac{d\mathbf{Y}_d}{dt} = \frac{1}{16\pi^2} \mathbf{Y}_d [\; & 3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_u{}^\dagger\mathbf{Y}_u + Tr\{3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} \\
& - (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \;] \;.
\end{aligned}
\tag{4.2.1}
$$

where $\mathbf{Y}_{u,d,e}$ are the matrices of Yukawa couplings. Demanding that their crossing point be within the unification region determined by the gauge couplings allows one to constrain M_t. This yields an upper and lower bound for M_t which nevertheless is fairly restrictive.

There is a threshold at M_{SUSY}. Here the matching condition is the naive one of simple continuity due to the lack of knowledge about the superparticle spectrum. The scale is taken to be variable to account for this ignorance.

4.3 One Light Higgs Limit

The simplest implementation of supersymmetry is considered and the couplings are run above M_{SUSY} to one loop. The MSSM is assumed above

M_{SUSY}, and a model with a single light Higgs scalar below it. This is done by integrating out one linear combination of the two doublets at M_{SUSY}, thereby leaving the orthogonal combination in the Standard Model regime as the "Higgs doublet"

$$\Phi_{(SM)} = \Phi_d \cos\beta + \tilde{\Phi}_u \sin\beta , \qquad (4.3.1)$$

where $\tilde{\Phi} = i\tau_2\Phi^*$, and where $\tan\beta$ is also the ratio of the two vacuum expectation values (v_u/v_d) in the limit under consideration. This sets boundary conditions on the Yukawa couplings at M_{SUSY}. Furthermore, in this approximation the quartic self-coupling of the surviving Higgs at the SUSY scale is given by

$$\lambda(M_{SUSY}) = \frac{1}{4}(g_1^2 + g_2^2)\cos^2(2\beta) . \qquad (4.3.2)$$

This correlates the mixing angle with the quartic coupling and thereby gives a value for the physical Higgs mass, M_H. Using the experimental limits on the M_H further constrains some of the results. By using the renormalization group one takes into account radiative corrections to the light Higgs mass [54] and hence relax the tree level upper bound, $M_H \sim M_Z$ [55].

4.4 Initial Data

The bounds on M_t and M_H are determined by probing these masses dependence on β. In SUSY-$SU(5)$, $\tan\beta$ is constrained to be larger than one in the one light Higgs limit. It seems natural to require that $y_t \geq y_b$ up to the unification scale [56], thereby yielding an upper bound on $\tan\beta$. The initial values at M_Z for the gauge couplings are taken to be [1,57]

$$\alpha_1 = 0.016887 \pm 0.000040 ,$$

$$\alpha_2 = 0.03322 \pm 0.00025 , \qquad (4.4.1)$$

$$\alpha_3 = 0.109^{+0.004}_{-0.005} ,$$

where GUT normalization for α_1 is used. The following set of four quark running masses defined at 1 GeV by the Particle Data book [21] are used $m_u = 5.6$ MeV, $m_d = 9.9$ MeV, $m_s = 199$ MeV, and $m_c = 1.35$ GeV. For the bottom mass, the Gasser and Leutwyler bottom mass value of 5.3 GeV at 1 GeV is used which translates into a physical mass of $M_b = 4.6$ GeV [26]. To probe the dependence of the results on M_b, the case $M_b = 5$ GeV is also studied, this is the typical value obtained from potential model fits for bottom quark bound states [58]. The effect of varying M_{SUSY} is also investigated. Given the values of the gauge couplings, unification holds for SUSY scales up to 8.9 TeV and as low as M_W. For empirical reasons solutions below M_W were not investigated.

4.5 Analysis and Results

The inclusion of supersymmetry collapses the GUT triangle. This is illustrated in Fig. 12 taking $M_{SUSY} = 1$ TeV. As mentioned above, a range $M_W \lesssim M_{SUSY} \lesssim 9$ TeV will achieve unification within one sigma error. From Fig. 13 (the magnified unification region of Fig. 12), one determines that the lower end scale, M_{GUT}^L, of the unification region corresponds to an α_3 value of 0.104 at M_Z, while the higher end scale, M_{GUT}^H, corresponds to a value of 0.108 at M_Z for α_3. It is found that the unification region is insensitive to the range of top, bottom, and Higgs masses considered. In the analysis of the bounds for M_t, the values for α_1 and α_2 are chosen to be the central values since their associated experimental uncertainties are less significant than for α_3. Demanding that y_b and y_τ cross at M_{GUT}^L and taking $\alpha_3 = 0.104$ then sets a lower bound on M_t. Correspondingly, demanding that y_b and y_τ cross

at M_{GUT}^{H} and taking $\alpha_3 = 0.108$ yields an upper bound on M_t. These bounds are found for each possible value of β.

Figure 14 shows the upper and lower bound curves for both M_t and M_H as a function of β and for $M_{SUSY} = 1$ TeV and $M_b = 4.6$ GeV. When applicable the current experimental limit of 38 GeV on the light supersymmetric neutral Higgs mass [59] is used to determine the lowest possible M_t value consistent with the model. It is found that $139 \leq M_t \leq 194$ GeV and $44 \leq M_H \leq 120$ GeV. The sensitivity of these results on M_{SUSY} is investigated in the range, 1.0 ± 0.5 TeV. It is found that the bounds on M_t are not modified, but the upper bound on the Higgs is changed to 125 GeV, and the lower bound drops below the experimental lower bound.

For $M_b = 5.0$ GeV, an overall decrease in the top and Higgs mass bounds is observed $116 \leq M_t \leq 181$ GeV, $M_H \leq 111$ GeV. Varying M_{SUSY} as above modifies the respective bounds. The top mass lower and upper bounds become 113 and 119 GeV, respectively. The upper bound on M_H changes to 115 GeV. The results of the analysis are displayed for the extreme case, $M_{SUSY} = 8.9$ TeV, in Fig. 15, with $M_b = 4.6$ GeV. This only significantly changes the upper bound on M_H to 144 GeV compared to the $M_{SUSY} = 1$ TeV case.

y_t has also been run up to the unification region and compared with y_b and y_τ to see what the angle β must be for these three couplings to meet [60], as in an $SO(10)$ or E_6 model [61,62] with a minimal Higgs structure. It is clear that this angle is precisely the upper bound on β as described earlier. In Fig. 16 y_t/y_b is displayed at the GUT scale as a function of $\tan \beta$ for $M_{SUSY} = 1$ TeV and for the two bottom masses considered. If one demands that the ratio be one, one can determine the mixing angles for the low and high ends of the unification region. Then going back to Fig. 14, one finds as expected a much

tighter bound on the masses of the top and of the Higgs. Indeed, for $M_b = 4.6$ GeV, one has $49.40 \leq \tan \beta \leq 54.98$, which yields $162 \leq M_t \leq 176$ GeV and $106 \leq M_H \leq 111$ GeV. When $M_b = 5.0$ GeV, one obtains $31.23 \leq \tan \beta \leq 41.18$, which gives $116 \leq M_t \leq 147$ GeV and $93 \leq M_H \leq 101$ GeV.

The four mass relations of Section 3.7 were also studied in the context of the MSSM [46]. In this case, these relations can all be satisfied at the scale of gauge coupling unification. However, for this to be true, several things must happen: first V_{cb} must be larger than its presently measured central value of 0.043; second the top quark mass must be around 190 GeV (if it is lighter, then agreement dictates that V_{cb} should be larger still); third the Higgs boson mass should be around 120 GeV. An analysis which recently appeared in the literature has reached similar conclusions [63].

4.6 Comments

To improve on this analysis, one should implement the supersymmetric two loop beta functions and the corresponding thresholds. The effects of soft SUSY breaking terms should be investigated. Also, all the supersymmetric particles have been integrated out at the same scale. It would be interesting to study the effect of lifting this restriction. It should be noted that the bounds on the top mass are very similar to those of Ref. 56, although the physics is very different. These issues will be addressed in a subsequent chapter. However, given the relative crudeness of the approximations made here, it is remarkable that the experimental bounds on the ρ–parameter were satisfied. A sign which gives credence to the program.

Table 4: Particle Quantum Numbers in the MSSM									
	$\hat{Q}$	$\hat{\bar{u}}$	$\hat{\bar{d}}$	$\hat{L}$	$\hat{\bar{e}}$	$\hat{\Phi}_u$	$\hat{\Phi}_d$	$\hat{g}^A$	$\hat{W}^a$	$\hat{B}$
$U(1)$	$+\frac{1}{6}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$SU(2)$	2	1	1	2	1	2	2	1	3	1
$SU(3)$	3	$\bar{3}$	$\bar{3}$	1	1	1	1	8	1	1
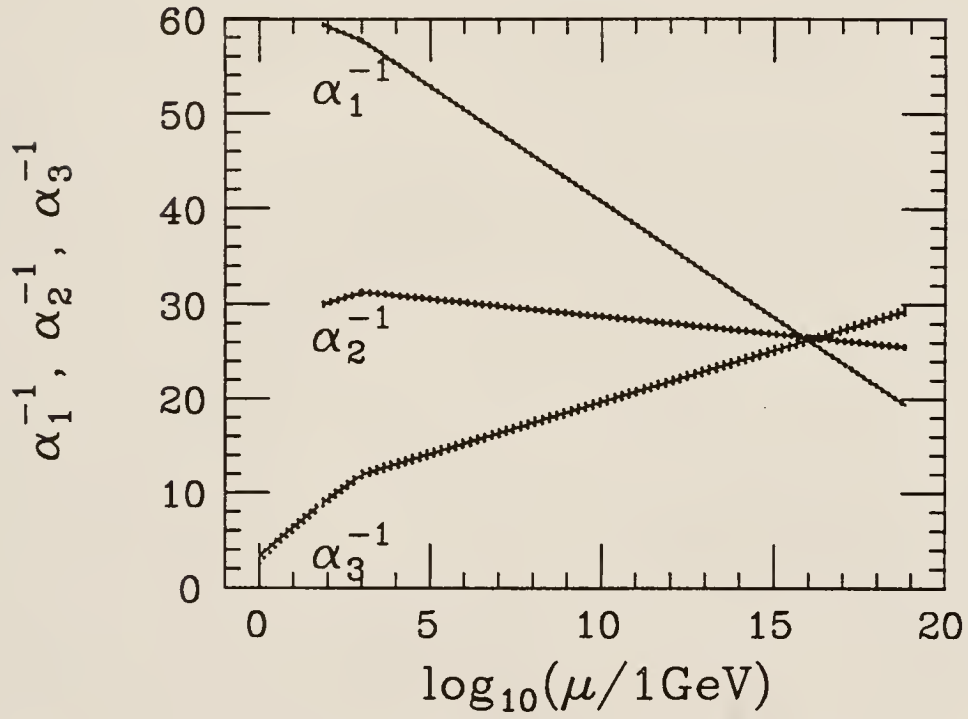
Figure 12. Plot of the running of the inverse couplings. The dotted lines above and below the solid lines represent the experimental error for each coupling.
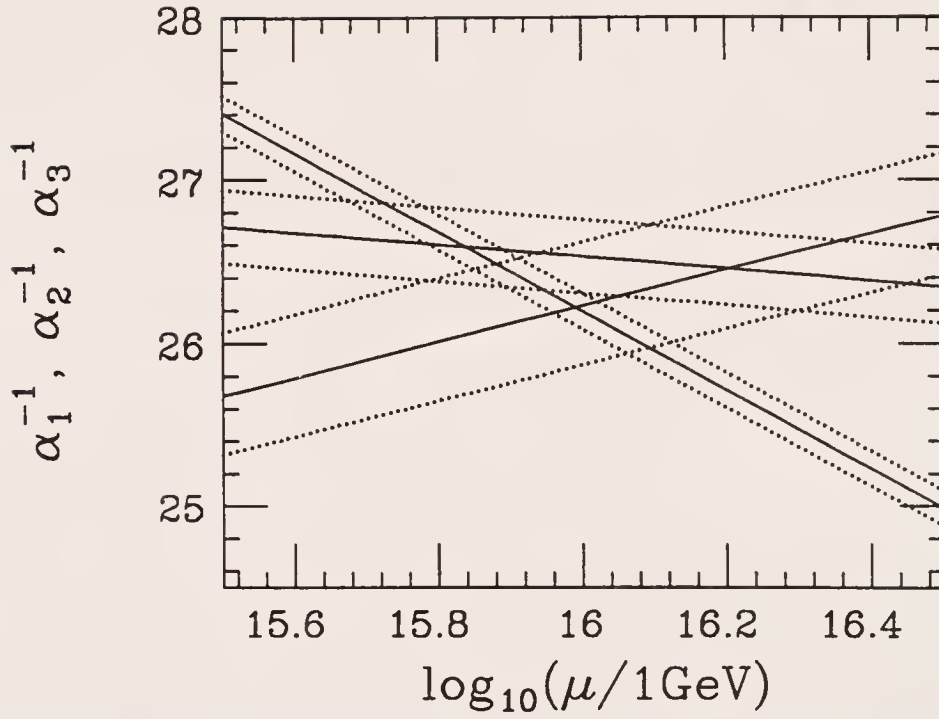
Figure 13. The plot depicts a blow-up of the area around the unifi-
cation point. (Note the small region where all three cou-
plings intersect. This region reduced to a point when
M_{SUSY} = 8.9 TeV and was non-existent above that
scale.)
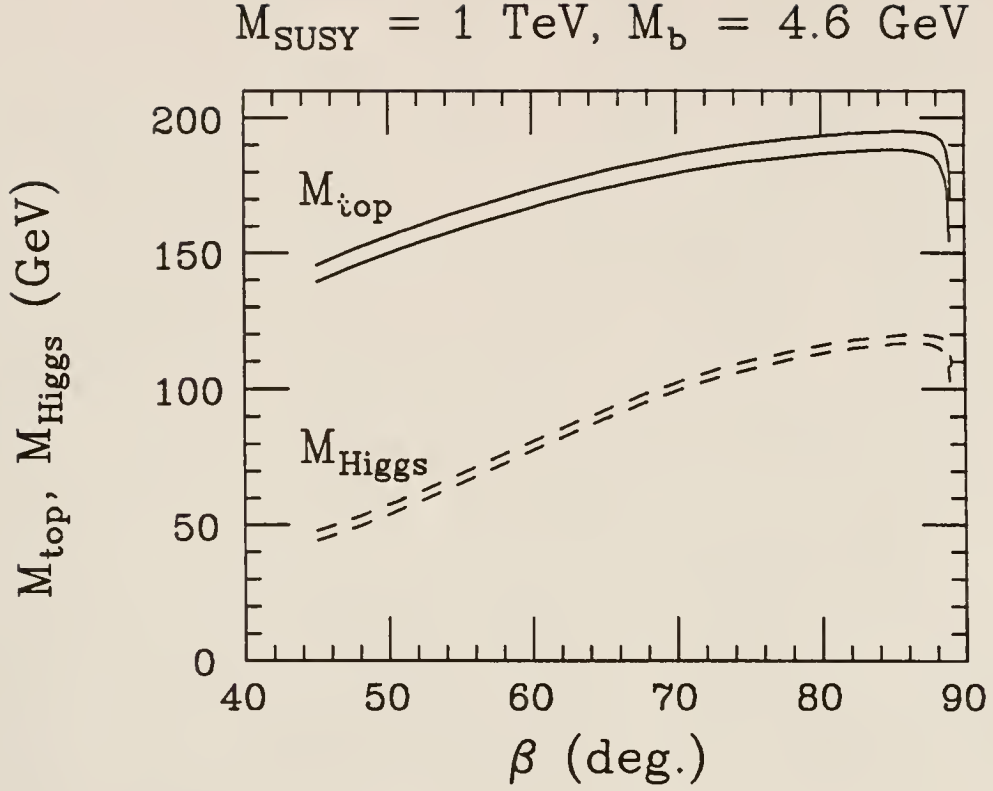
$$M_{SUSY} = 1 \ TeV, \ M_b = 4.6 \ GeV$$

Figure 14. Plot of the top quark mass, M_t, and of the Higgs mass, M_H, as a function of the mixing angle β for the highest value of α_3 (high curves) and the lowest value of α_3 (low curves) consistent with unification as *per* Fig. 12.

$$M_{SUSY} = 8.9 \text{ TeV}, \; M_b = 4.6 \text{ GeV}$$
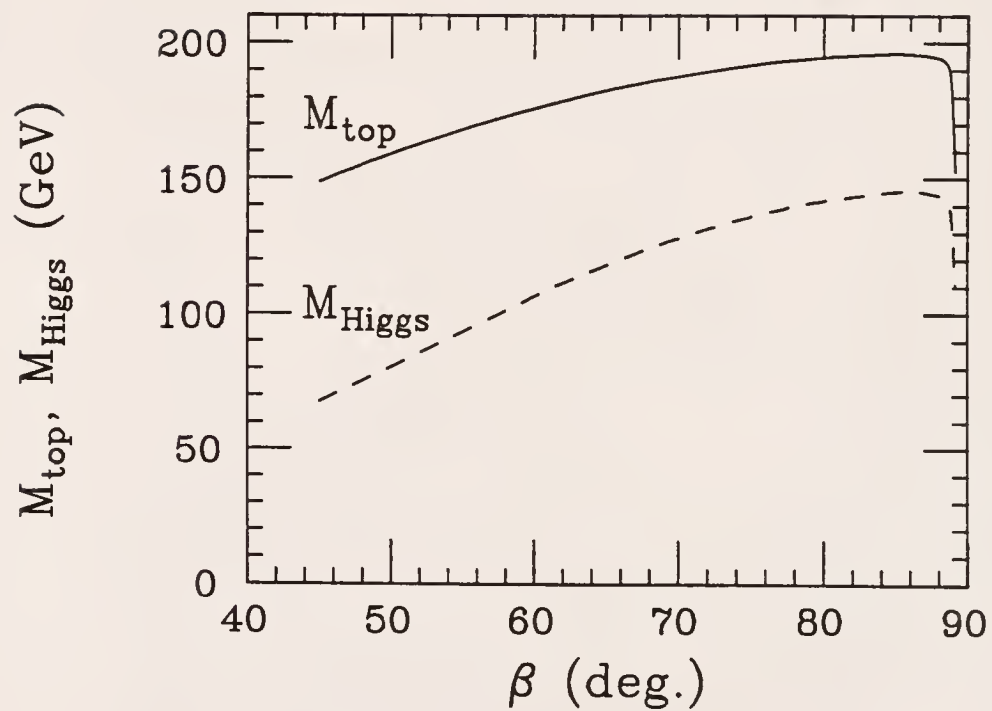
Figure 15. Same as Fig. 14 for $M_{SUSY} = 8.9$ TeV and $M_b = 4.6$ GeV.

$M_{SUSY} = 1$ TeV

$M_b = 4.6$ GeV

$M_b = 5.0$ GeV

$y_t(M_{GUT})/y_b(M_{GUT})$

$\tan\beta$

Figure 16. Plot of the ratio of the top to bottom Yukawas, y_t/y_b, for two different bottom masses (solid and dashed curves) as a function of $\tan\beta$ for the highest value of α_3 (high curves) and the lowest value of α_3 (low curves) consistent with unification as *per* Fig. 12.

CHAPTER 5
SOFT SYMMETRY BREAKING

5.1 Minimal Low Energy Supergravity Model

Since no super particles have been observed experimentally, supersymmetry, if truly present in nature, must be broken. One way to accomplish this breaking is to couple the standard model to $N = 1$ su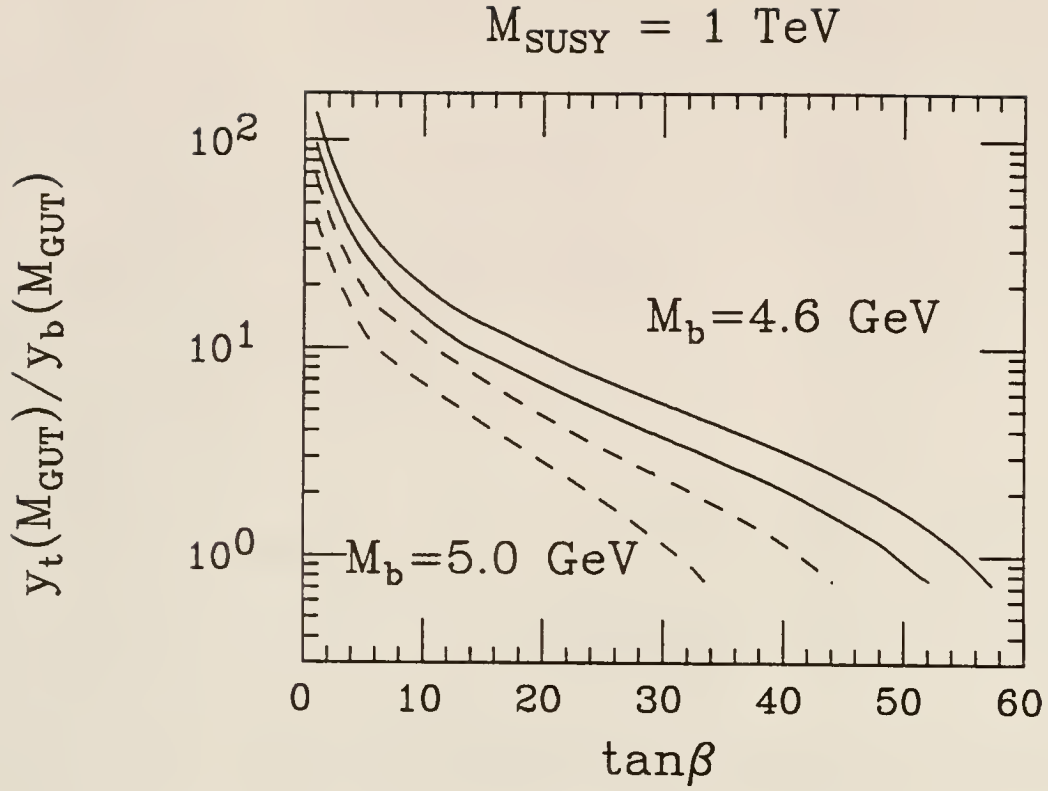pergravity (SUGRA). In the minimal low energy supergravity model considered, supersymmetry is explicitly broken by the addition of supergravity induced soft terms (including gaugino mass terms)

$$
\begin{aligned}
V_{soft} &= m_{\phi_u}^2 \Phi_u^\dagger \Phi_u + m_{\phi_d}^2 \Phi_d^\dagger \Phi_d + B\tilde{\mu}(\Phi_u \Phi_d + h.c.) \\
&+ \sum_i (\, m_{Q_i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{L_i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{u_i}^2 \tilde{\bar{u}}_i^\dagger \tilde{\bar{u}}_i + m_{d_i}^2 \tilde{\bar{d}}_i^\dagger \tilde{\bar{d}}_i + m_{e_i}^2 \tilde{\bar{e}}_i^\dagger \tilde{\bar{e}}_i \,) \\
&+ \sum_{i,j} (\, A_u^{ij} Y_u^{ij} \tilde{\bar{u}}_i \Phi_u \tilde{Q}_j + A_d^{ij} Y_d^{ij} \tilde{\bar{d}}_i \Phi_d \tilde{Q}_j + A_e^{ij} Y_e^{ij} \tilde{\bar{e}}_i \Phi_d \tilde{L}_j + h.c. \,) \, , \\
V_{gaugino} &= \frac{1}{2} \sum_{l=1}^{3} M_l \lambda_l \lambda_l + h.c. \, ,
\end{aligned}
$$

$$(5.1.1)$$

where $V_{gaugino}$ is the Majorana mass terms for the gaugino fields, λ_l (suppressing the group index), corresponding to $U(1)$, $SU(2)$, and $SU(3)$, respectively. From the supersymmetry algebra, one deduces that spontaneous symmetry breaking occurs if and only if the vacuum energy is not zero. In global supersymmetric theories, the scalar potential is a sum of F and D terms. Supersymmetry is spontaneously broken if either the vacuum values of the F term ([64]) or D term ([65]) are non-zero. A consequence of the spontaneous

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symmetry breaking is a massless fermion in analogy with the breaking of an ordinary global symmetry. We will assume that the spontaneous breaking of the local N=1 supersymmetry is communicated to the "visible" sector by weak gravitational interactions from some "hidden" sector. Spontaneous symmetry breaking in supergravity occurs via the super-Higgs mechanism. The goldstone fermion, or goldstino, associated with the breaking of global supersymmetry is eaten by the gravitino thereby providing it with a mass. This spontaneous symmetry breaking of supergravity manifests itself at low energy as explicit soft breaking terms of global supersymmetry. This leads to a common (gravitino) mass, m_0, for the scalars of the model and masses, M_l, for the gauginos at the GUT scale, M_X. By assuming gauge coupling unification, we can take the three gaugino masses equal. Furthermore, the trilinear soft couplings A_u^{ij}, A_d^{ij}, and A_e^{ij} are all equal to a common value A_0. The bilinear soft coupling B_0 may be related to A_0 ($B_0 = A_0 - m_0$), if the SUGRA model has only canonical kinetic terms for the chiral superfields. This scenario is to be contrasted with one in which general soft breaking terms are added *ad hoc* to the Lagrangian. In the most general case, there are sixty-three soft symmetry breaking parameters. Their number can be reduced by invoking certain symmetries such as flavor and family blindness. In either case, all of these couplings will evolve to different values under the renormalization group. The complete scalar potential appears as

$$V = V_F + V_D + V_{soft} , \tag{5.1.2}$$

where V_F contains the potential contributions from the F-terms

$$\begin{aligned}
V_F = &|\tilde{\bar{u}}\mathbf{Y}_u\tilde{Q} + \breve{\mu}\Phi_d|^2 + |\tilde{\bar{d}}\mathbf{Y}_d\tilde{Q} + \tilde{\bar{e}}\mathbf{Y}_e\tilde{L} + \breve{\mu}\Phi_u|^2 \\
&+ |\mathbf{Y}_u\tilde{Q}\Phi_u|^2 + |\mathbf{Y}_d\tilde{Q}\Phi_d + \mathbf{Y}_e\tilde{L}\Phi_d|^2 \\
&+ |\tilde{\bar{u}}\mathbf{Y}_u\Phi_u + \tilde{\bar{d}}\mathbf{Y}_d\Phi_d|^2 + |\tilde{\bar{e}}\mathbf{Y}_e\Phi_d|^2 ,
\end{aligned} \tag{5.1.3}$$

and V_D contains the potential contributions from the D-terms

$$
\begin{aligned}
V_D = \frac{g'^2}{2}\big(& \frac{1}{6}\tilde{Q}^\dagger\tilde{Q} - \frac{2}{3}\tilde{u}^\dagger\tilde{u} + \frac{1}{3}\tilde{d}^\dagger\tilde{d} - \frac{1}{2}\tilde{L}^\dagger\tilde{L} + \tilde{e}^\dagger\tilde{e} + \frac{1}{2}\Phi_u^\dagger\Phi_u - \frac{1}{2}\Phi_d^\dagger\Phi_d \big)^2 \\
& + \frac{g_2^2}{8}\big(\tilde{Q}^\dagger\vec{\tau}\tilde{Q} + \tilde{L}^\dagger\vec{\tau}\tilde{L} + \Phi_u^\dagger\vec{\tau}\Phi_u + \Phi_d^\dagger\vec{\tau}\Phi_d \big)^2 \\
& + \frac{g_3^2}{8}\big(\tilde{Q}^\dagger\vec{\lambda}\tilde{Q} - \tilde{u}^\dagger\vec{\lambda}^*\tilde{u} - \tilde{d}^\dagger\vec{\lambda}^*\tilde{d} \big)^2 \, ,
\end{aligned}
$$

$$(5.1.4)$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the $SU(2)$ Pauli matrices and $\vec{\lambda} = (\lambda_1, \ldots, \lambda_8)$ are the Gell-Mann matrices. In general, one must impose constraints on the parameters to avoid charge and color breaking minima in the scalar potential. Some necessary constraints have been formulated, such as

$$
\begin{aligned}
A_U^2 &< 3(m_Q^2 + m_u^2 + m_{\phi_u}^2) \, , \\
A_D^2 &< 3(m_Q^2 + m_d^2 + m_{\phi_d}^2) \, , \\
A_E^2 &< 3(m_L^2 + m_e^2 + m_{\phi_d}^2) \, .
\end{aligned}
$$

$$(5.1.5)$$

However, these are in general neither sufficient nor indeed always necessary [66]. They involve very specific assumptions about the spontaneous symmetry breaking.

5.2 Radiative Electro-Weak Breaking

An appealing feature of the models being considering is that they can lead to the breaking of the electro-weak symmetry radiatively. The one loop potential responsible for the breaking is

$$
V_{1-loop}(\mu) = V_{tree}(\mu) + \triangle V_1(\mu) \, ,
$$

$$(5.2.1)$$

where

$$
\begin{aligned}
\triangle V_1(\mu) &= \frac{1}{64\pi^2}\text{Str}\{\mathcal{M}^4(\ln\frac{\mathcal{M}^2}{\mu^2} - \frac{3}{2})\} \\
&= \frac{1}{64\pi^2}\sum_p (-1)^{2s_p}(2s_p + 1)m_p^4(\ln\frac{m_p^2}{\mu^2} - \frac{3}{2}) \, ,
\end{aligned}
$$

$$(5.2.2)$$

where $\mathcal{M}^2$ is the field dependent squared mass matrix of the model and m_p is the eigenvalue mass of the pth particle of spin s_p. The tree level potential is

$$V_{tree}(\mu) = m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u + m_3^2 (\ \Phi_u \Phi_d + h.c.\ )$$
$$+ \frac{g'^2}{8} (\ \Phi_u^\dagger \Phi_u - \Phi_d^\dagger \Phi_d\ )^2 + \frac{g_2^2}{8} (\ \Phi_u^\dagger \vec{\tau} \Phi_u + \Phi_d^\dagger \vec{\tau} \Phi_d\ )^2 \quad (5.2.3)$$

where

$$m_1^2 = m_{\phi_d}^2 + \check{\mu}^2\ ,$$
$$m_2^2 = m_{\phi_u}^2 + \check{\mu}^2\ , \quad (5.2.4)$$
$$m_3^2 = B\check{\mu}\ .$$

Minimization gives

$$\frac{1}{2} m_Z^2 = \frac{\overline{m}_1^2 - \overline{m}_2^2 \tan^2 \beta}{\tan^2 \beta - 1}, \quad (5.2.5)$$

where $m_Z^2 = (g'^2 + g_2^2) v^2 / 2$, $v^2 = v_u^2 + v_d^2$ and

$$B\check{\mu} = \frac{1}{2} (\overline{m}_1^2 + \overline{m}_2^2) \sin 2\beta\ , \quad (5.2.6)$$

where $\tan \beta = v_u / v_d$, also

$$\overline{m}_i^2 = m_i^2 + \frac{\partial \Delta V_1}{\partial v_i^2}\ , \quad (5.2.7)$$

where $v_1 = v_d$, $v_2 = v_u$ and

$$\frac{\partial \Delta V_1}{\partial v_i^2} = \frac{1}{32\pi^2} \sum_p (-1)^{2s_p} (2s_p + 1) m_p^2 (\ln \frac{m_p^2}{\mu^2} - 1) \frac{\partial m_p^2}{\partial v_i^2}\ . \quad (5.2.8)$$

The parameters of the potential are taken as running ones, that is, they vary with scale as dictated by the renormalization group. Because the one loop correction to the potential is not negligible at all scales, an appropriate scale must be chosen if the tree level formulas are to be valid [67]. Indeed, the tree level analysis may lead to incorrect conclusions about the regions of parameter space that yield electroweak breaking and consistent scenarios. When ΔV_1 is included, the value of μ is not critical as long as μ is in the neighborhood of M_W.

Although, as stated, the tree level results cannot always be trusted, one can get some idea under what conditions electro-weak breaking occurs. The renormalization group evolution of $m^2_{\phi_u}$ (see Appendix C) can be such that it turns negative at low energies if the top quark mass is large enough, whereas $m^2_{\phi_d}$ runs positive. From Eq. (5.2.3), the scale at which this occurs is set by the condition

$$m^2_1(\mu_b)\, m^2_2(\mu_b) - m^4_3(\mu_b) = 0 \ . \tag{5.2.9}$$

If the free parameters are adjusted properly, then the correct value of the Z^0 mass ($M_Z = 91.17$ GeV) can be achieved. At tree level there is another critical scale that must be considered. Again from Eq. (5.2.3), it is evident that the potential becomes unbounded from below along the equal field (neutral components) direction, if

$$m^2_1(\mu_s) + m^2_2(\mu_s) < 2m^2_3(\mu_s) \ . \tag{5.2.10}$$

Since $m^2_1 m^2_2 - m^4_3 \geq 0$ implies $m^2_1 + m^2_2 \geq 2m^2_3$, condition ((5.2.10)) can only occur at scales lower than condition ((5.2.9)), so $\mu_s < \mu_b$. As mentioned above, when working with the tree level potential an appropriate scale to minimize it must be used at which one loop corrections may be safely neglected. Gamberini et al. [67] give a prescription for the choice of this scale.

In the present work, the one loop corrections are incorporated. Contributions from the third generation are included, that is, the top-stop, bottom-sbottom, and tau-stau. The one loop effective potential should be constant against the renormalization group to this order. The Z^0 mass is chosen as the scale at which to evaluate the minimization conditions Eqs. (5.2.9)-(5.2.10). Equation (5.2.5) can be written

$$\check{\mu}^2(M_Z) = \frac{\overline{m}^2_{\phi_d} - \overline{m}^2_{\phi_u} \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m^2_Z \ , \tag{5.2.11}$$

where $\overline{m}^2_{\phi_{u,d}} = m^2_{\phi_{u,d}} + \partial \Delta V_1 / \partial v^2_{u,d}$ and used to solve for $\breve{\mu}(M_Z)$ (see Numerical Procedure Section) given the value of all the relevant parameters at M_Z. A choice for the sign of $\breve{\mu}$ must be made ($\breve{\mu}$ is multiplicatively renormalized; see Appendix C). It is evident that no adequate minimum exists if the parameters are such that $\breve{\mu}^2 < 0$.

5.3 Sfermion Masses

The mass matrices for scalar matter are constructed from Eq. (5.1.2). For example, in the up squark sector the relevant mass matrix appears as

$$\mathcal{M}^2_{\tilde{u}} = \begin{pmatrix} M^2_{L_i L_j} & M^2_{L_i R_j} \\ M^2_{R_i L_j} & M^2_{L_i R_j} \end{pmatrix} \tag{5.3.1}$$

where $i, j = 1, 2, 3$ are flavor indices and

$$M^2_{L_i L_j} = m^2_{Q_i} \delta_{ij} + v^2_u (\mathbf{Y}^\dagger_u \mathbf{Y}_u)_{ij} - \frac{1}{2}(v^2_d - v^2_u)(Y(u_L)g'^2 - T_3(u_L)g^2_2)\delta_{ij} \ ,$$

$$M^2_{R_i R_j} = m^2_{u_i} \delta_{ij} + v^2_u (\mathbf{Y}^\dagger_u \mathbf{Y}_u)_{ij} - \frac{1}{2}(v^2_d - v^2_u)(Y(u_R)g'^2)\delta_{ij} \ ,$$

$$M^2_{R_i L_j} = \breve{\mu} v_d Y^{ij}_u + v_u A^{ij}_u Y^{ij}_u \ ,$$

$$M^2_{L_i R_j} = M^{2*}_{R_j L_i} \ .$$

$$\tag{5.3.2}$$

Note in Table 4 that $Y = Q - T_3$ in the adopted notation. Similar matrices follow for the other sfermions. These mass formulas as well as the ones to follow are given in terms of running parameters. The domain of validity of these formulas is at low energies ($\sim M_Z$) with the parameters taking on their renormalization group evolved values at this scale.

5.4 Higgs Masses

If the following notation is employed

$$\Phi_1 = \begin{pmatrix} \phi^0_1 \\ \phi^-_1 \end{pmatrix} \ , \Phi_2 = \begin{pmatrix} \phi^+_2 \\ \phi^0_2 \end{pmatrix} \ ,$$

then the physical masses of the Higgs at tree level are calculated from the following three matrices

$$\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(\Im\phi_i^0)\partial(\Im\phi_j^0)} = \frac{1}{2}M_A^2 \sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix}$$

$$\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(\Re\phi_i^0)\partial(\Re\phi_j^0)} = \frac{1}{2}M_A^2 \sin 2\beta \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix}$$

$$+ \frac{1}{2}m_Z^2 \sin 2\beta \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix}$$

$$\frac{\partial^2 V_{tree}}{\partial(\phi_i^-)\partial(\phi_j^+)} = \frac{1}{2}M_{H\pm}^2 \sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix}$$

(5.4.1)

where $M_A^2 = m_1^2 + m_2^2$, $M_{H\pm}^2 = M_A^2 + m_W^2$, and $m_W^2 = g_2^2 v^2/2$. The eigenvalues for the first matrix are 0 corresponding to the Goldstone boson and M_A^2 corresponding to the CP odd scalar. The second matrix gives the masses of the light and heavy Higgs bosons

$$M_{H,h}^2 = \frac{1}{2}[(M_A^2 + m_Z^2) \pm \sqrt{(M_A^2 + m_Z^2)^2 - 4M_A^2 m_Z^2 \cos^2 2\beta}] . \qquad (5.4.2)$$

This tree level result predicts $M_h < M_Z$. One loop calculations show that this need not be the case [68]. The third matrix has eigenvalues 0 and $M_{H\pm}^2$ corresponding to a massless, charged Goldstone boson and a charged scalar. Including Eq. (5.2.2) in the calculations leads to corresponding one loop versions of these masses [69,70].

5.5 Chargino Masses

The following four terms contribute to the chargino masses

$$-i\sqrt{2}g_2\Phi_u^\dagger\frac{\tau^i}{2}\tilde{\Phi}_u\tilde{W}^i - i\sqrt{2}g_2\Phi_d^\dagger\frac{\tau^i}{2}\tilde{\Phi}_d\tilde{W}^i - \breve{\mu}\tilde{\Phi}_u\tilde{\Phi}_d + \frac{1}{2}M_2\tilde{W}^i\tilde{W}^i + h.c. . \quad (5.5.1)$$

The first two terms are the supersymmetric Yukawa-gauge terms. Letting $\lambda^{\pm} = (\tilde{W}_2 \pm i\tilde{W}_1)/\sqrt{2}$, the mass matrix follows

$$
\begin{pmatrix} \lambda^+ & \tilde{\phi}_u^+ & \lambda^- & \tilde{\phi}_d^- \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & 0 & M_2 & -g_2 v_d \\ 0 & 0 & g_2 v_u & -\breve{\mu} \\ M_2 & g_2 v_u & 0 & 0 \\ -g_2 v_d & -\breve{\mu} & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda^+ \\ \tilde{\phi}_u^+ \\ \lambda^- \\ \tilde{\phi}_d^- \end{pmatrix} . \tag{5.5.2}
$$

Diagonalization yields two charged Dirac fermions with masses

$$
M^2_{\tilde{H}_{\pm},\tilde{h}_{\pm}} = \frac{1}{2}[(M_2^2 + \breve{\mu}^2 + 2m_W^2) \\ \pm \sqrt{(M_2^2 + \breve{\mu}^2 + 2m_W^2)^2 - 4(M_2\breve{\mu} - m_W^2 \sin 2\beta)^2}] . \tag{5.5.3}
$$

5.6 Neutralino Masses

Contributing to the neutralino masses are the terms in Eq. (5.5.1) and

$$
-ig'\sqrt{2}\Phi_u^{\dagger}(+\frac{1}{2})\tilde{\Phi}_u\tilde{B} - ig'\sqrt{2}\Phi_d^{\dagger}(-\frac{1}{2})\tilde{\Phi}_d\tilde{B} + \frac{1}{2}M_1\tilde{B}\tilde{B} . \tag{5.6.1}
$$

The neutralino mass matrix follows

$$
\begin{pmatrix} i\tilde{B} & i\tilde{W}_3 & \tilde{\phi}_d^0 & \tilde{\phi}_u^0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} -M_1 & 0 & \frac{g'v_d}{\sqrt{2}} & -\frac{g'v_u}{\sqrt{2}} \\ 0 & -M_2 & -\frac{g_2 v_d}{\sqrt{2}} & \frac{g_2 v_u}{\sqrt{2}} \\ \frac{g'v_d}{\sqrt{2}} & -\frac{g_2 v_d}{\sqrt{2}} & 0 & \breve{\mu} \\ -\frac{g'v_u}{\sqrt{2}} & \frac{g_2 v_u}{\sqrt{2}} & \breve{\mu} & 0 \end{pmatrix} \begin{pmatrix} i\tilde{B} \\ i\tilde{W}_3 \\ \tilde{\phi}_d^0 \\ \tilde{\phi}_u^0 \end{pmatrix} . \tag{5.6.2}
$$

5.7 Boundary Conditions at M_X

In this work, the modified minimal subtraction scheme ($\overline{\text{MS}}$) of renormalization is employed. The parameters of the Lagrangian are not in general equal to any corresponding physical constant. For example, in the case of masses, except for those of the bottom and top quark (see Eq. (3.4.1)), all other physical masses will be determined from their corresponding running masses by the simpler relation

$$
M = m(\mu)\Big|_{\mu=M} . \tag{5.7.1}
$$

The renormalization group β functions of the gauge and Yukawa couplings have been calculated to two loops without making any approximations in the Yukawa sector of the model. These have been included in Appendix C.

Because SUGRA models make certain simplifying predictions about the soft parameters at the unification (Planck) scale, the evolution of the renormalization group equations is initiated at this scale. It has been demonstrated that the introduction of supersymmetry leads to gauge coupling unification at approximately $\sim 10^{16}$ GeV. Therefore one takes $M_X = 10^{16}$ GeV.

At the unification scale, M_X, all scalars have a common mass

$$
\begin{aligned}
m_{Q_i}(M_X) &= m_{u_i}(M_X) = m_{d_i}(M_X) = m_{L_i}(M_X) \\
&= m_{e_i}(M_X) = m_{\phi_u}(M_X) = m_{\phi_d}(M_X) = m_0 ,
\end{aligned}
\tag{5.7.2}
$$

as do the gauginos

$$
M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{\frac{1}{2}} .
\tag{5.7.3}
$$

The trilinear soft scalar couplings will be taken diagonal and equal at M_X

$$
A_u^{ij}(M_X) = A_d^{ij}(M_X) = A_e^{ij}(M_X) = A_0 .
\tag{5.7.4}
$$

Also the bilinear soft scalar coupling and the mixing mass at M_X are given by

$$
B(M_X) = B_0, \quad \breve{\mu}(M_X) = \breve{\mu}_0 .
\tag{5.7.5}
$$

Furthermore, to constrain the parameter space, the bottom and tau masses will be assumed equal at M_X

$$
m_b(M_X) = m_\tau(M_X) .
\tag{5.7.6}
$$

Also for the sake of CPU time, we will take the masses of the two lightest families to be zero in all the runs.

5.8 Numerical Procedure

There are seven free parameters in the model considered. These are A_0, B_0, m_0, $m_{1/2}$, $\breve{\mu}_0$, $\tan\beta$, and m_t. The two minimization constraints (5.2.5) and (5.2.6) reduce this set to five, which are taken to be A_0, m_0, $m_{1/2}$, $\tan\beta$, and m_t. In the present framework, B_0 and $\breve{\mu}_0$ will be determined using the numerical solutions routines described in Appendix E in conjunction with the minimization of the one loop effective potential at M_Z. Minimization at M_Z will give $B(M_Z)$ and $\breve{\mu}(M_Z)$. To arrive at B_0 and $\breve{\mu}_0$ (their corresponding values at M_X), the solution routine are employed as follows. A guess for B_0 and $\breve{\mu}_0$ is made at M_X and then the parameters of the model are run to M_Z at which scale the evolved value of B is compared to the minimization output value for B at M_Z. The same is done for $\breve{\mu}$. If the compared values agree to some set accuracy, then B_0 and $\breve{\mu}_0$ are the required values. Other analyses that also extract $B(M_Z)$ and $\breve{\mu}(M_Z)$ simply evolve these two parameters via their renormalization group equations back to M_X to find B_0 and $\breve{\mu}_0$ relying on their near decoupling from the full set of renormalization group equations. Notice from Eq. (5.2.11) that a choice must be made for the sign of $\breve{\mu}$. To constrain the parameter space further, the bottom quark and tau lepton masses will be taken equal at M_X. This equality is a characteristic of many SUSY-GUTs. This constrains the model to four free parameters, A_0, m_0, $m_{1/2}$, and $\tan\beta$. Demanding that $m_b(M_X) = m_\tau(M_X)$ and achieving the correct physical masses for the bottom quark and tau lepton fixes the mass of the top quark which affects the evolution of the bottom Yukawa significantly. Gauge coupling unification shall be assumed, an assumption which appears reasonable when one considers SUSY models with SUSY breaking scales $\lesssim 10$

TeV. The solution routines could be used to find the precise (and similar) values of α_1, α_2, and α_3 at M_X that will evolve to the experimentally known values at M_Z, however this increases the CPU time considerably. Therefore some precision shall be sacrificed in their M_Z values by taking them exactly equal at M_X. This is already a theoretical oversimplification since one does not expect the gauge couplings to be exactly equal due to threshold effects at the GUT scale. It is found that for all cases studied, the common value $\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_3^{-1}(M_X) = 25.31$ leads to errors no bigger than 1%, 5%, and 10% in $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, and $\alpha_3(M_Z)$, respectively. This is not so bad considering that the (combined experimental and theoretical) errors on $\alpha_3(M_Z)$ from some processes can be as large as this.

It is well known that there is a fine tuning problem inherent in the radiatively induced electro-weak models. For certain values of the parameters, the top quark mass must be tuned to an "unnaturally" high degree of accuracy to achieve the correct value of M_Z. This problem is generally handled by rejecting models that require "too much" tuning. The amount of tuning is usually defined quite arbitrarily. The usual procedure is to define fine tuning parameters

$$c_i = |\frac{x_i^2}{M_Z^2} \frac{\partial M_Z^2}{\partial x_i^2}|\,, \tag{5.8.1}$$

where x_i are parameters of the theory such as m_0, $m_{1/2}$, $\tilde{\mu}$, or m_t. One then demands that the c_i be less than some chosen value that is typically taken to be 10. The differences in using the tree level *vs.* one loop effective potential were analyzed to some extent. The results agree generally with those of Ref. 67. Moreover it is found that the fine tuning problem is exacerbated in the tree level analysis. The basis for the "theoretical" fine tuning problem can be seen,

if one makes some simplifying assumptions, in the dependence of M_W on the top quark Yukawa coupling y_t [71]

$$M_W \sim M_X e^{-1/y_t^2} . \qquad (5.8.2)$$

In the tree level analysis, one encounters another fine tuning problem. The vacuum expectation value coming from the minimization conditions changes rapidly from 0 to infinity over the interval (μ_s, μ_b). Using the prescription of Ref. 67 for the scale $\hat{\mu}$ at which to adequately minimize the tree level potential, find $v(\hat{\mu})$, and thereby arrive at a value for M_Z, one finds that although a small variation in $y_t(M_X)$ may lead to a small variation in μ_b, the steepness in the tree level vacuum expectation value will lead to a large variation in the value of $v(\hat{\mu})$ and therefore in M_Z. Hence, in the tree level analysis, solutions which may be within the bounds of the "theoretical" fine tuning may nevertheless display a fine tuning aspect because of this "tree level" fine tuning problem. The use of the one loop effective potential levels off the vev around the M_Z scale. The vevs depend on scale through wave function renormalization effects which are never large as can be seen from the form of the renormalization group equations for the vevs in Appendix C.

In the present framework, solutions shall also be rejected based on fine tuning considerations, however the present method differs somewhat from the usual one in that it is incorporated in the solution routine described above. The routine is an iterative one which determines the convergence properties of the solution which reflect an inherent fine tuning. If the convergence is too slow, the solution will be rejected. Effectively any solution which the computer cannot pinpoint within an allotted number of iterations is rejected. Given values for A_0, m_0, $m_{1/2}$, $\tan \beta$, and $\text{sign}(\mu)$, the solution routines search for

the values of $v(M_X)$, $m_{b,\tau}(M_X)$, $m_t(M_X)$, B_0, and $\breve{\mu}_0$. The process by which B_0 and $\breve{\mu}_0$ are found was described above. The remaining three parameters are determined similarly. The routine makes a guess for $v(M_X)$, $m_{b,\tau}(M_X)$, and $m_t(M_X)$, then the full renormalization group equations are evolved to 1 GeV calculating superparticle threshold masses in the process and minimizing the one loop effective potential at M_Z. The merits of the guess for $v(M_X)$, $m_{b,\tau}(M_X)$, and $m_t(M_X)$ is assessed by comparing the resulting values of M_Z, $m_\tau(1 \text{ GeV})$, and $m_b(1 \text{ GeV})$ with the expected ones. The process is iterated until the correct values are achieved to within some tolerance.

5.9 Thresholds

In the minimal low energy supergravity model being considered, the super particle spectrum is no longer degenerate as in the simple global supersymmetry model in which all the super particles are given a common mass, M_{SUSY}. In the simple case, one makes one course correction in the renormalization group evolution at M_{SUSY}. In the model with soft symmetry breaking, the nondegenerate spectrum should lead to various course corrections at the super particle mass thresholds. To this end, the renormalization group β functions must be cast in a new form which makes the implementation of the thresholds effects (albeit naive) evident (see Appendix B). Since the $\overline{\text{MS}}$ renormalization group equations are mass independent, particle thresholds must be handled using the decoupling theorem [39], and each super particle mass has associated with it a boundary between two effective theories. Above a particular mass threshold the associated particle is present in the effective theory, below the threshold the particle is absent.

The simplest way to incorporate this is to (naively) treat the thresholds as steps in the particle content of the renormalization group β functions. This method is not always entirely adequate. For example, in the case of the $SU(2)$ gauge coupling there will be scales in the integration process at which there are effectively a half integer number of doublets using this method. Nevertheless, this method should yield the correct, general behavior of the evolution. It is a simple means of implementing the smearing effects of the non-degenerate super particle spectrum. The determination of the spectrum of masses is done without iteration as is common in other analyses. The method employed in this work deduces the physical masses by solving the equation $m(\mu) = \mu$ for each superparticle in the process of evolving from M_X to 1 GeV. The usual iterative method requires several runs to find a consistent solution.

5.9 Analysis and Results

The tremendous computing task involved in analyzing the full parameter space of the soft symmetry breaking models, using the methods described as designed, would be far too time consuming given the available computing facilities. Therefore, in the following analysis, some simplifications will be made in the procedural method. First, only the heaviest family of quarks and leptons will have non-zero mass. Second, as stated previously, the value of the strong coupling at M_Z will be allowed to vary from its central value of .113 by at most 10%. This translates into a similar error in the bottom quark mass. Third, the allotted number of Runge-Kutta steps, involved in numerically integrating the renormalization group equations, will be cut down to ~ 100.

The present analysis will restrict itself to a subclass of soft symmetry breaking models that have two or three soft parameters equal to zero at M_X. One

such class follows from the no-scale model [72] and has $A_0 = m_0 = 0$. The strict no-scale model has $A_0 = m_0 = B_0 = 0$. Another class coming from string derived models has $A_0 = B_0 = 0$. Since B_0 is not a free parameter in the procedure adopted in this work (see Section 5.8), $B_0 = 0$ results must be inferred.

Because the GUT inspired constraint, $m_b(M_X) = m_\tau(M_X)$, is enforced in this analysis, the results will depend on the mass of the bottom quark. Most results will be reported for the case $m_b(1\text{ GeV}) = 6.00$ GeV, but lower mass (5.70 GeV) and higher mass (6.33 GeV) cases were also studied. The running value of 6.00 GeV for $m_b(1\text{ GeV})$ corresponds to a physical bottom mass $M_b = 4.85 \pm .15$ GeV, with the uncertainty coming from the error in the strong coupling, as discussed above. The results of Chapter 4, indicate that the top quark mass predictions increase with decreasing bottom mass, and the present analysis corroborates this.

Given the no-scale or string-inspired cases, the phase space is explored by setting $A_0 = m_0 = 0$ or $A_0 = 0$, respectively, and coarse graining the remaining hyperslice. $\tan\beta$ was most commonly coarse grained as 2, 5, and 10. The mass of the lowest supersymmetric particle (LSP), when it is a neutralino, is observed to be correlated with the value of $m_{1/2}$. Cosmological considerations indicate that the LSP must be neutral and colorless and have a mass less than ~ 200 GeV. Points in parameter space that lead to LSPs other than neutralinos with masses less than ~ 200 GeV (or sneutrinos) are rejected.

Figure 17 shows a slice of the available phase space, in the no-scale case, plotted against the top quark mass. The present unofficial bound on the top quark mass is ~ 120 GeV. The figure indicates that the top quark cannot have a mass greater than ~ 132 GeV in this model, if $m_b(1\text{ GeV}) = 6.00$ GeV

(This upper bound is raised to ~ 160 GeV, if $m_b(1 \text{ GeV}) = 5.70$ GeV, and the model is ruled out, if $m_b(1 \text{ GeV}) = 6.33$ GeV). The figure also indicates that $190 \text{ GeV} \lesssim m_{1/2} \lesssim 265$ GeV; however, nothing significant can be said about the value of $\tan\beta$. The points to the right of the allowed region were found to lead to charged LSPs ($\tilde{\tau}_R$) and are not displayed. Points to the left of the region do not lead to electro-weak breaking. Furthermore, as Figs. 14 and 15 indicated, the top quark mass does not grow monotonically with $\tan\beta$, rather it reaches a maximum for some high $\tan\beta$ value then decreases. For the choice $\text{sign}(\mu) = -$, the top quark mass upper bound falls below the experimental limit, so this case is ruled out.

Analysis of the available data indicates that the strict no-scale case lowers the top quark mass upper bound slightly to ~ 128. However, there is an allowed range of $3 - 9$ for $\tan\beta$. Figure 18 is a plot of the evolution of the left-handed and right-handed stop, sbottom, stau, and gaugino soft mass parameters for the particular strict no-scale case $m_{1/2} = 240$ GeV, $\tan\beta = 8.3$, $\text{sign}(\mu) = +$. The resulting spectrum of super particle masses is presented in Table 5. The LSP is a neutralino in this case and has a mass of 92 GeV. The top quark mass is just above the experimental limit at 126 GeV, and the Higgs boson mass is 77 GeV.

The strict no-scale case is a special case of the string inspired one. therefore the 128 GeV top quark mass upper bound is not expected to decrease but rather to increase in the "stringy" case. The data indicate that top quark masses as high as ~ 150 GeV are possible. Finally, a representative stringy scenario is presented. Figure 19 displays the variation of M_t and B_0 with $\tan\beta$ for the slice of parameter space with $A_0 = 0$, $m_0 = 100$ GeV, and $m_{1/2} = 250$ GeV. The scenario inferred from the figure has $M_t = 130$ and $\tan\beta = 7.8$.

Table 5	
A_0	0
m_0	0
$m_{1/2}$	240
$\tilde{\mu}_0$	171
$\tan\beta$	8.3
$\text{sign}(\mu)$	$+$
M_t	126
$\tilde{d}$	$473 - 506$
$\tilde{u}$	$390 - 532$
$\tilde{e}$	$94 - 176$
$\tilde{\nu}_L$	180
$\tilde{\gamma}$	92
$\tilde{Z}, \tilde{W}$	153, 147
$\tilde{g}$	557
$\tilde{H}^0$	213, 271
$\tilde{H}^\pm$	270
h, H	77, 158
$H^\pm, A$	235, 222

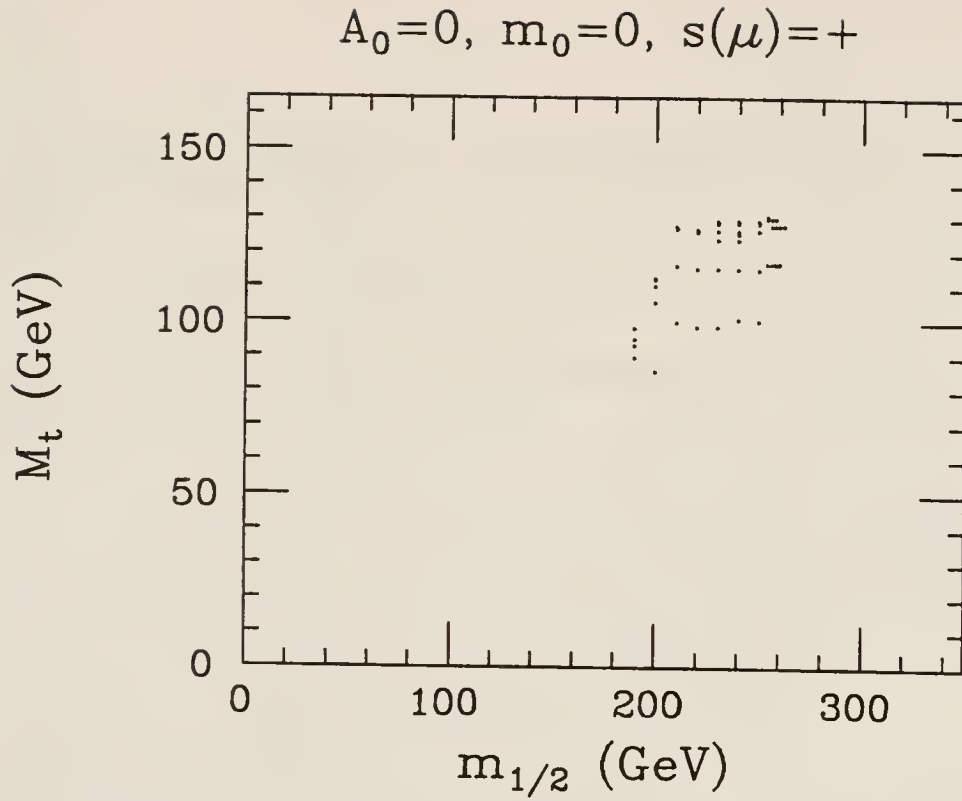$$A_0=0, \quad m_0=0, \quad s(\mu)=+$$

Figure 17. Slice of parameter space in no-scale case displaying M_t against the common gaugino mass $m_{1/2}$. The points represent radiative electro-weak breaking solutions with $m_b = m_\tau$ at M_X and a neutralino LSP with mass $\lesssim 200$ GeV.
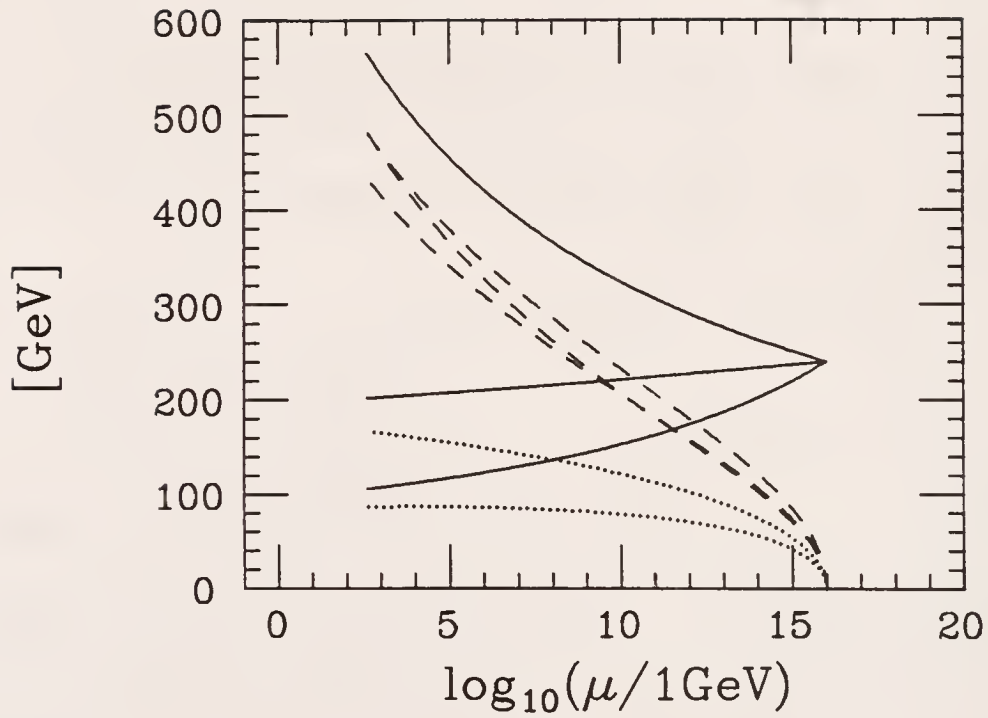
Figure 18. Evolution of squark (dashes), slepton (dots), and gaugino (solid) soft mass terms for $A_0 = 0$, $m_0 = 0$, $m_{1/2} = 240$ GeV, and $\tan\beta = 8.3$.
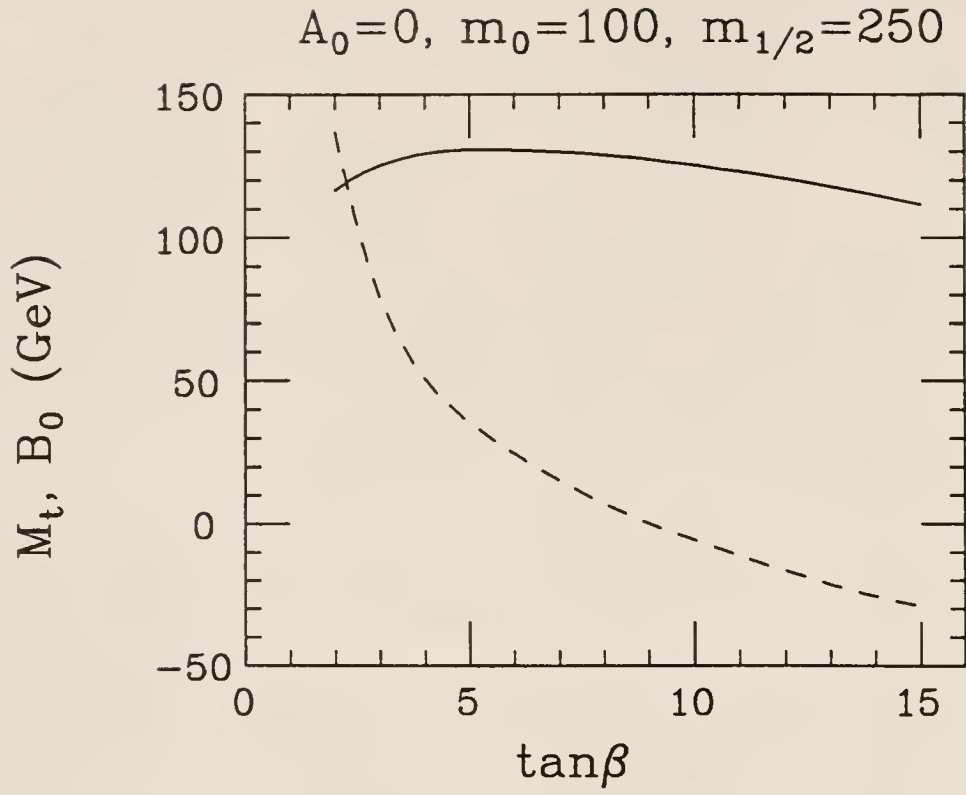
Figure 19. M_t (solid) and B_0 (dashes) plotted against $\tan \beta$ for $A_0 = 0$, $m_0 = 100$ GeV, and $m_{1/2} = 250$ GeV to infer the $B_0 = 0$ case.

CHAPTER 6
CONCLUSIONS

This work has studied, using one and two loop renormalization group β functions, both the standard model and its minimal supersymmetric extension. The parameters of these models, *i.e.*, the gauge couplings, the quark, squark, lepton, and slepton masses, the Yukawa sector mixing angles and phase, the scalar quartic coupling, and soft symmetry breaking parameters, were run over scales ranging from 1 GeV to Planck mass. The aspects of the standard model and its minimal supersymmetric extension were reviewed.

In the standard model case, thresholds effects were served well by naive step functions. The more sophisticated implementation using one loop matching functions did not significantly improve on the step function method. The difference between two loops and one loop represented a more significant effect, albeit sometimes small. Plots exhibiting these different features for all the parameters of the Standard Model were included. Gauge coupling unification was shown to fail, and some interesting mass and mixing angle relations were considered, but failed to hold simultaneously at a common scale.

In the supersymmetric case, in which gauge coupling unification is achieved, bounds on the top quark and Higgs boson masses were determined using the $SU(5)$ inspired constraint that the bottom and tau masses be equal at the scale of unification. Remarkably, the top quark bounds were consistent with the ρ parameter bounds. In supersymmetric context, the mass relations could

all be accommodated at the scale of unification provided the top quark mass was high ~ 190.

Minimal low energy supergravity models were considered. They have the appealing feature that the electro-weak symmetry is radiatively broken for certain ranges of the soft breaking parameters. The study of specific models, with some soft parameter fixed, resulted in upper bounds for the top quark mass. No-scale models in which only gaugino masses provide global supersymmetry breaking yield top quarks with masses less than ~ 130 GeV. The results are sensitive to the value of the bottom quark mass. Lower bottom quark masses, within the experimental uncertainty, lead to higher top quark upper bounds. In these models, the ratio of vacuum expectation values of the two Higgs fields is expected to be larger than $\sim 72°$.

APPENDIX A
THE STANDARD MODEL β FUNCTIONS

In this appendix, the renormalization group β functions of the Standard Model are compiled [6]. These have appeared in one form or another in various sources. Effort has gone into confirming their validity through a comparative analysis of the literature. The main source is Ref. 7. Following their conventions,

$$\mathcal{L} = \overline{u}\mathbf{Y}_u\tilde{\Phi}^\dagger Q + \overline{d}\mathbf{Y}_d\Phi^\dagger Q + \overline{e}\mathbf{Y}_e\Phi^\dagger L + \text{h.c.} - \frac{1}{2}\lambda(\Phi^\dagger\Phi)^2 \ , \qquad (A.1)$$

where flavor indices have been suppressed, and where Q and L are the left-handed quark and lepton SU(2) doublets, respectively,

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \ , \ L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \ . \qquad (A.2)$$

Φ and $\tilde{\Phi}$ are the Higgs scalar doublet and its SU(2) conjugate

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \ , \ \tilde{\Phi} = i\tau_2\Phi \ . \qquad (A.3)$$

u_R, d_R, and e_R are the quark and lepton SU(2) singlets, and $\mathbf{Y}_{u,d,e}$ are the matrices of the up-type, down-type, and lepton-type Yukawa couplings.

The β functions for the gauge couplings are

$$\begin{aligned}
\frac{dg_l}{dt} = &-b_l\frac{g_l^3}{16\pi^2} - \sum_k b_{kl}\frac{g_k^2 g_l^3}{(16\pi^2)^2} \\
&- \frac{g_l^3}{(16\pi^2)^2}\text{Tr}\{C_{lu}\mathbf{Y}_u{}^\dagger\mathbf{Y}_u + C_{ld}\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + C_{le}\mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} \ ,
\end{aligned} \qquad (A.4)$$

where $t = \ln\mu$ and $l = 1, 2, 3$, corresponding to the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the Standard Model. The various coefficients are defined to be

$$b_1 = -\frac{4}{3}n_g - \frac{1}{10} ,$$
$$b_2 = \frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6} , \tag{A.5}$$
$$b_3 = 11 - \frac{4}{3}n_g ,$$

$$(b_{kl}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} - n_g \begin{pmatrix} \frac{19}{15} & \frac{1}{5} & \frac{11}{30} \\ \frac{3}{5} & \frac{49}{3} & \frac{3}{2} \\ \frac{44}{15} & 4 & \frac{76}{3} \end{pmatrix} - \begin{pmatrix} \frac{9}{50} & \frac{3}{10} & 0 \\ \frac{9}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{A.6}$$

and

$$(C_{lf}) = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix} , \text{ with } f = u , d , e , \tag{A.7}$$

with $n_g = \frac{1}{2}n_{fl}$.

In the Yukawa sector the β functions are

$$\frac{d\mathbf{Y}_{u,d,e}}{dt} = \mathbf{Y}_{u,d,e}\left(\frac{1}{16\pi^2}\beta^{(1)}_{u,d,e} + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{u,d,e}\right) , \tag{A.8}$$

where the one-loop contributions are given by

$$\beta^{(1)}_u = \frac{3}{2}(\mathbf{Y}_u{}^\dagger\mathbf{Y}_u - \mathbf{Y}_d{}^\dagger\mathbf{Y}_d) + Y_2(S) - (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2) ,$$
$$\beta^{(1)}_d = \frac{3}{2}(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d - \mathbf{Y}_u{}^\dagger\mathbf{Y}_u) + Y_2(S) - (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2) , \tag{A.9}$$
$$\beta^{(1)}_e = \frac{3}{2}\mathbf{Y}_e{}^\dagger\mathbf{Y}_e + Y_2(S) - \frac{9}{4}(g_1^2 + g_2^2) ,$$

with

$$Y_2(S) = \text{Tr}\{3\mathbf{Y}_u{}^\dagger\mathbf{Y}_u + 3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} , \tag{A.10}$$

and the two-loop contributions are given by

$$
\begin{aligned}
\beta_u^{(2)} = \; & \frac{3}{2}(Y_u{}^\dagger Y_u)^2 - Y_u{}^\dagger Y_u Y_d{}^\dagger Y_d - \frac{1}{4}Y_d{}^\dagger Y_d Y_u{}^\dagger Y_u + \frac{11}{4}(Y_d{}^\dagger Y_d)^2 \\
& + Y_2(S)(\frac{5}{4}Y_d{}^\dagger Y_d - \frac{9}{4}Y_u{}^\dagger Y_u) - \chi_4(S) + \frac{3}{2}\lambda^2 - 2\lambda(3Y_u{}^\dagger Y_u + Y_d{}^\dagger Y_d) \\
& + (\frac{223}{80}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2)Y_u{}^\dagger Y_u - (\frac{43}{80}g_1^2 - \frac{9}{16}g_2^2 + 16g_3^2)Y_d{}^\dagger Y_d \\
& + \frac{5}{2}Y_4(S) + (\frac{9}{200} + \frac{29}{45}n_g)g_1^4 - \frac{9}{20}g_1^2 g_2^2 + \frac{19}{15}g_1^2 g_3^2 - (\frac{35}{4} - n_g)g_2^4 + 9g_2^2 g_3^2 \\
& - (\frac{404}{3} - \frac{80}{9}n_g)g_3^4 \; , \\[4pt]
\beta_d^{(2)} = \; & \frac{3}{2}(Y_d{}^\dagger Y_d)^2 - Y_d{}^\dagger Y_d Y_u{}^\dagger Y_u - \frac{1}{4}Y_u{}^\dagger Y_u Y_d{}^\dagger Y_d + \frac{11}{4}(Y_u{}^\dagger Y_u)^2 \\
& + Y_2(S)(\frac{5}{4}Y_u{}^\dagger Y_u - \frac{9}{4}Y_d{}^\dagger Y_d) - \chi_4(S) + \frac{3}{2}\lambda^2 - 2\lambda(3Y_d{}^\dagger Y_d + Y_u{}^\dagger Y_u) \\
& + (\frac{187}{80}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2)Y_d{}^\dagger Y_d - (\frac{79}{80}g_1^2 - \frac{9}{16}g_2^2 + 16g_3^2)Y_u{}^\dagger Y_u \\
& + \frac{5}{2}Y_4(S) - (\frac{29}{200} + \frac{1}{45}n_g)g_1^4 - \frac{27}{20}g_1^2 g_2^2 + \frac{31}{15}g_1^2 g_3^2 - (\frac{35}{4} - n_g)g_2^4 \\
& + 9g_2^2 g_3^2 - (\frac{404}{3} - \frac{80}{9}n_g)g_3^4 \; , \\[4pt]
\beta_e^{(2)} = \; & \frac{3}{2}(Y_e{}^\dagger Y_e)^2 - \frac{9}{4}Y_2(S)Y_e{}^\dagger Y_e - \chi_4(S) + \frac{3}{2}\lambda^2 - 6\lambda Y_e{}^\dagger Y_e \\
& + (\frac{387}{80}g_1^2 + \frac{135}{15}g_2^2)Y_e{}^\dagger Y_e + \frac{5}{2}Y_4(S) + (\frac{51}{200} + \frac{11}{5}n_g)g_1^4 + \frac{27}{20}g_1^2 g_2^2 \\
& - (\frac{35}{4} - n_g)g_2^4 \; ,
\end{aligned}
$$

$$\text{(A.11)}$$

with

$$
\begin{aligned}
Y_4(S) = \; & (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\mathrm{Tr}\{Y_u{}^\dagger Y_u\} \\
& + (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\mathrm{Tr}\{Y_d{}^\dagger Y_d\} + \frac{3}{4}(g_1^2 + g_2^2)\mathrm{Tr}\{Y_e{}^\dagger Y_e\} \; ,
\end{aligned}
$$

$$\text{(A.12)}$$

and

$$
\begin{aligned}
\chi_4(S) = \; & \frac{9}{4}\mathrm{Tr}\{3(Y_u{}^\dagger Y_u)^2 + 3(Y_d{}^\dagger Y_d)^2 + (Y_e{}^\dagger Y_e)^2 \\
& - \frac{2}{3}Y_u{}^\dagger Y_u Y_d{}^\dagger Y_d\} \; .
\end{aligned}
$$

$$\text{(A.13)}$$

In the Higgs sector, the β functions for the quartic coupling and the vacuum expectation of the scalar field are presented. Here a discrepancy in the one-loop contribution to the quartic coupling of Ref. 7 is corrected

$$
\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\lambda^{(2)} \; .
$$

$$\text{(A.14)}$$

where the one-loop contribution is given by

$$\beta_\lambda^{(1)} = 12\lambda^2 - (\frac{9}{5}g_1^2 + 9g_2^2)\lambda + \frac{9}{4}(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4)$$
$$+ 4Y_2(S)\lambda - 4H(S) , \tag{A.15}$$

with

$$H(S) = \text{Tr}\{3(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 + 3(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2\} , \tag{A.16}$$

and the two-loop contribution is given by (discrepancies found by Ford et $al.$ [73] in Machacek and Vaughn [7] are corrected),

$$\beta_\lambda^{(2)} = -78\lambda^3 + 18(\frac{3}{5}g_1^2 + 3g_2^2)\lambda^2 - [\ (\frac{313}{8} - 10n_g)g_2^4 - \frac{117}{20}g_1^2 g_2^2$$
$$+ \frac{9}{25}(\frac{229}{4} + \frac{50}{9}n_g)g_1^4\ ]\lambda + (\frac{497}{8} - 8n_g)g_2^6 - \frac{3}{5}(\frac{97}{24} + \frac{8}{3}n_g)g_1^2 g_2^4$$
$$- \frac{9}{25}(\frac{239}{24} + \frac{40}{9}n_g)g_1^4 g_2^2 - \frac{27}{125}(\frac{59}{24} + \frac{40}{9}n_g)g_1^6$$
$$- 64g_3^2\text{Tr}\{(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 + (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2\}$$
$$- \frac{8}{5}g_1^2\text{Tr}\{2(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 - (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + 3(\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2\}$$
$$- \frac{3}{2}g_2^4 Y_4(S) + 10\lambda[\ (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\} \tag{A.17}$$
$$+ (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\text{Tr}\{\mathbf{Y}_d^\dagger \mathbf{Y}_d\} + \frac{3}{4}(g_1^2 + g_2^2)\text{Tr}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e\}\ ]$$
$$+ \frac{3}{5}g_1^2[\ (-\frac{57}{10}g_1^2 + 21g_2^2)\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u\} + (\frac{3}{2}g_1^2 + 9g_2^2)\text{Tr}\{\mathbf{Y}_d^\dagger \mathbf{Y}_d\}$$
$$+ (-\frac{15}{2}g_1^2 + 11g_2^2)\text{Tr}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e\}\ ] - 24\lambda^2 Y_2(S) - \lambda H(S)$$
$$+ 6\lambda\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d\} + 20\text{Tr}\{3(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^3 + 3(\mathbf{Y}_d^\dagger \mathbf{Y}_d)^3$$
$$+ (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^3\} - 12\text{Tr}\{\mathbf{Y}_u^\dagger \mathbf{Y}_u(\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)\mathbf{Y}_d^\dagger \mathbf{Y}_d\} .$$

The β function for the vacuum expectation value of the scalar field is

$$\frac{d\ln v}{dt} = \frac{1}{16\pi^2}\gamma^{(1)} + \frac{1}{(16\pi^2)^2}\gamma^{(2)} , \tag{A.18}$$

where the one-loop contribution is given by

$$\gamma^{(1)} = \frac{9}{4}(\frac{1}{5}g_1^2 + g_2^2) - Y_2(S) , \tag{A.19}$$

and the two-loop contribution is given by

$$
\begin{aligned}
\gamma^{(2)} = -\frac{3}{2}\lambda^2 - \frac{5}{2}Y_4(S) + \chi_4(S) \\
-(\frac{93}{800} + \frac{1}{2}n_g)g_1^4 + (\frac{511}{32} - \frac{5}{2}n_g)g_2^4 - \frac{27}{80}g_1^2 g_2^2 .
\end{aligned}
\tag{A.20}
$$

These expressions were arrived at using the general formulas provided in Ref. 7

for the anomalous dimension of the scalar field, choosing the Landau gauge.

In the low energy regime the effective theory is $SU(3)_C \times U(1)_{EM}$. The

general formula of Ref. 74 is used to arrive at the β functions for the respective

gauge couplings

$$
\begin{aligned}
\frac{dg_3}{dt} = [\frac{2}{3}(n_u + n_d) - 11]\frac{g_3^3}{(4\pi)^2} + [\frac{38}{3}(n_u + n_d) - 102]\frac{g_3^5}{(4\pi)^4} \\
+ [\frac{8}{9}n_u + \frac{2}{9}n_d]\frac{g_3^3 e^2}{(4\pi)^4} \\
+ [\frac{5033}{18}(n_u + n_d) - \frac{325}{54}(n_u + n_d)^2 - \frac{2857}{2}]\frac{g_3^7}{(4\pi)^6} ,
\end{aligned}
\tag{A.21}
$$

and

$$
\begin{aligned}
\frac{de}{dt} = [\frac{16}{9}n_u + \frac{4}{9}n_d + \frac{4}{3}n_l]\frac{e^3}{(4\pi)^2} + [\frac{64}{27}n_u + \frac{4}{27}n_d + 4n_l]\frac{e^5}{(4\pi)^4} \\
+ [\frac{64}{9}n_u + \frac{16}{9}n_d]\frac{e^3 g_3^2}{(4\pi)^4} ,
\end{aligned}
\tag{A.22}
$$

where n_u, n_d, and n_l are the number of up-type quarks, down-type quarks, and

leptons, respectively. In Eq. (A.21), the three-loop pure QCD contribution to

the β function of g_3 have also been included [75].

For the evolution of the fermion masses, Ref. 76 is used. It is known that

there is an error in their printed formula [77]. This typographical error is

found and corrected. The calculation of the two-loop contribution to the mass

anomalous dimension in QCD using the corrected formula agrees with the

result obtained in Ref. 77. Equation (2.26) of Ref. 76 should read

$$
\begin{aligned}
\gamma_{m_1}^{ij} = \delta_{ij}(\frac{17}{2}(C_{F_L}^i + C_{F_R}^j) - \frac{148}{3}C_{F_M}^i)C_A^j - 24C_{F_M}^i C_{F_M}^j + 12C_{F_M}^i(C_{F_L}^j + C_{F_R}^j) \\
- \frac{3}{2}(C_{F_L}^i C_{F_L}^j + C_{F_R}^i C_{F_R}^j) + T_R^i(\frac{32}{3}C_{F_M}^j - 2C_{F_L}^j - 2C_{F_R}^j)\delta_{ij} .
\end{aligned}
\tag{A.23}
$$

Using the corrected expression, the following mass anomalous dimension is computed. The fermion masses in the low energy theory then evolve as follows

$$\frac{dm}{dt} = \gamma_{(l,q)} m \, , \tag{A.24}$$

where the l and q refer to a particular lepton or quark, and where

$$\begin{aligned} \gamma_{(l,q)} &= \gamma_{(l,q)}^1 \frac{e^2}{(4\pi)^2} + \gamma_{(l,q)}^3 \frac{g_3^2}{(4\pi)^2} \\ &+ [\gamma_{(l,q)}^{11} e^4 + \gamma_{(l,q)}^{33} g_3^4 + 2\gamma_{(l,q)}^{13} e^2 g_3^2] \frac{1}{(4\pi)^4} \\ &+ \gamma_{(q)}^{333} \frac{g_3^6}{(4\pi)^6} \, . \end{aligned} \tag{A.25}$$

The superscripts 1 and 3 refer to the $U(1)_{EM}$ and $SU(3)_C$ contributions, respectively. Explicitly, the above coefficients are given by

$$\begin{aligned} \gamma_{(l,q)}^1 &= -6Q_{(l,q)}^2 \\ \gamma_{(l)}^3 &= 0 \\ \gamma_{(q)}^3 &= -8 \\ \gamma_{(l)}^{13} &= \gamma_{(l)}^{33} = 0 \\ \gamma_{(l,q)}^{11} &= -3Q_{(l,q)}^4 + [\frac{80}{9}n_u + \frac{20}{9}n_d + \frac{20}{3}n_l]Q_{(l,q)}^2 \\ \gamma_{(q)}^{13} &= -4Q_{(q)}^2 \\ \gamma_{(q)}^{33} &= -\frac{404}{3} + \frac{40}{9}(n_u + n_d) \\ \gamma_{(q)}^{333} &= \frac{2}{3}[\frac{140}{27}(n_u + n_d)^2 + (160\zeta(3) + \frac{2216}{9})(n_u + n_d) - 3747] \, , \end{aligned} \tag{A.26}$$

where $Q_{(l,q)}$ is the electric charge of a given lepton or quark, and $\zeta(3) = 1.2020\ldots$ is the Riemann zeta function evaluated at three. In the mass anomalous dimension for the quarks above, the three-loop pure QCD contribution $\gamma_{(q)}^{333}$ [75] have also been included.

APPENDIX B
CALCULATING THE MSSM β FUNCTIONS

In this appendix, some useful results in calculating the β function of the MSSM are included. Also some sample calculations are presented. Although, to one loop the MSSM β functions have appeared in the literature, one is not aware of the two loop ones' appearance except in some approximate form, such as keeping only the contribution of the heaviest family.

There are at least two ways to proceed. References [78,79] give general formulas valid to two loops to compute the β functions of gauge and Yukawa couplings in a supersymmetric gauge theory. These were used to calculate the two loop gauge and Yukawa coupling β functions appearing in the next appendix. Reference [7] gives formulas valid to two loops to compute the β functions of gauge, Yukawa, and scalar quartic couplings in a general gauge theory. For the purpose of including thresholds, this approach is more useful, and it is this manner that the following examples are done. The results of this second approach were checked against the first one, and they agreed (as they must).

Not all calculations are included, as this would be useless and not enlightening. However, it is hoped that the examples will give some flavor of the endeavor and that the results will prove useful to anyone wishing to pursue such calculations further.

B.1 Gauge Couplings

B.1.1 Group Invariants

Tables 6 and 7 will be useful in the calculations of the β functions of the MSSM. Also useful are the following group invariants

$$\mathbf{C_2}(R) = \mathbf{T}^A(R)\mathbf{T}^A(R) = C_2(R)\, \mathbf{I}(R) \; . \qquad \text{(B.1.1.1)}$$

Using this definition and

$$\text{Tr}\{\mathbf{T^A(R)T^B(R)}\} = \text{T}_2(\text{R})\delta^{AB} \qquad \text{(B.1.1.2)}$$

the following identity is derived

$$\text{Tr}\{\mathbf{C_2(R)}\} = d(G)T_2(R)$$

$$C_2(R)\text{Tr}\{\mathbf{I(R)}\} = rT_2(R) \qquad \text{(B.1.1.3)}$$

$$C_2(R)d(R) = T_2(R)r \; ,$$

where $r = d(G)$ is the rank of the group, and $\delta^{AA} = d(G)$. From this, it follows that

$$C_2(G) = T_2(G) \; . \qquad \text{(B.1.1.4)}$$

The Yukawa interactions arising in the SUSY Lagrangian can be written

$$\mathcal{L}_{Yukawa}^{SUSY} = -i\sqrt{2}g[\phi_i^* \, T_{ij}^A \, \psi_j\lambda^A - \overline{\psi}_j \, T_{ji}^A \, \phi_i\overline{\lambda}^A] \; , \qquad \text{(B.1.1.5)}$$

where $\mathbf{T}^A$ are the group generators, and overlined fermi (Grassmann) fields are right-handed. Now make the correspondence

$$Y_{ij}^a \xrightarrow{SUSY} \tilde{\mathbf{Y}}_{\phi(i)j}^a = -i\sqrt{2}gT_{ij}^A \; ,$$

$$\tilde{\mathbf{Y}}_{j\phi(i)}^a = +i\sqrt{2}gT_{ji}^A \; . \qquad \text{(B.1.1.6)}$$

Written this way, the i (i.e., $\phi(i)$) subscript represents the ith group component of the scalar, and the j represents the group component of the fermion. The i

encodes the scalar information in SUSY whereas the a does so in the notation of Ref. 7. When taking a Yukawa term from Ref. 7 and "supersymmetrizing" it, the trace need not represent a trace unless all scalar indices (i.e., the superscript on the $\check{Y}$'s) are contracted. For example, in the one-loop contribution to the scalar anomalous dimension

$$
\begin{aligned}
\mathrm{Tr}\{\check{Y}^a\check{Y}^{b\dagger}\} &= [-i\sqrt{2}gT^A_{ik}][i\sqrt{2}gT^A_{kj}] \\
&= 2g^2(\mathbf{T}^A\mathbf{T}^A)_{ij} \\
&= 2g^2 C_{2ij} \\
&= 2g^2 C_2(S)\delta_{ij}
\end{aligned}
\tag{B.1.1.7}
$$

Now to compute $Y_4(F)$.

$$
\begin{aligned}
\mathrm{Tr}\{C_2(\mathbf{F})\check{Y}^a\check{Y}^{a\dagger}\} &= 2g^2 C_2(F)\mathrm{Tr}\{\ \mathbf{I}\ \mathbf{T}^B\mathbf{T}^B\} \\
&= 2g^2 C_2(F)\mathrm{Tr}\{\mathbf{T}^B\mathbf{T}^C\}\delta^{CB} \\
&= 2g^2 C_2(F)T_2(R)\delta^{BC}\delta^{CB} \\
&= 2g^2 d(G)C_2(F)T_2(R)
\end{aligned}
\tag{B.1.1.8}
$$

$$
\begin{aligned}
Y_4(F) &= \frac{1}{d(G)}\mathrm{Tr}\{C_2(\mathbf{F})\check{Y}^a\check{Y}^{a\dagger}\} \\
&= 2g^2 C_2(F)T_2(R)\ .
\end{aligned}
\tag{B.1.1.9}
$$

These results may be used to derive the SUSY gauge beta function. In the general (but single simple group G) case, at one loop

$$
(4\pi)^2\beta^{(1)}/g^3 = \frac{2}{3}T_2(F) + \frac{1}{3}T_2(S) - \frac{11}{3}C_2(G)\ ,
\tag{B.1.1.10}
$$

where F, S, and G stand for the fermion, scalar, and adjoint representations, respectively. To go to the SUSY from, the following correspondences are used

$$
\begin{aligned}
F &\xrightarrow{SUSY} R + G\ , \\
S &\xrightarrow{SUSY} R\ .
\end{aligned}
\tag{B.1.1.11}
$$

Whence

$$(4\pi)^2 \beta_{SUSY}^{(1)}/g^3 = \frac{2}{3}[T_2(R) + C_2(G)] + \frac{1}{3}T_2(R) - \frac{11}{3}C_2(G)$$

$$= T_2(R) - 3C_2(G) \ . \tag{B.1.1.12}$$

At two loops, the only extra complication is a Yukawa contribution in the general case that must be supersymmetrized as discussed above

$$(4\pi)^4 \beta^{(2)}/g^5 = [\frac{10}{3}C_2(G) + 2C_2(F)]T_2(F) + [\frac{2}{3}C_2(G) + 4C_2(S)]T(S)$$

$$- \frac{34}{3}C_2(G)^2 - Y_4(F) \ . \tag{B.1.1.13}$$

Applying the stated correspondences yields

$$(4\pi)^2 \beta_{SUSY}^{(2)}/g^5 = \frac{10}{3}C_2(G)[T_2(R) + T_2(G)] + \frac{2}{3}C_2(G)T_2(R)$$

$$+ 2[C_2(R)T_2(R) + C_2(G)T_2(G)] + 4C_2(R)T_2(R)$$

$$- \frac{34}{3}C_2(G)^2 - [Y_4(R) + Y_4(G)]$$

$$= 4C_2(G)T_2(R) + 6C_2(R)T_2(R) - 6C_2(G)^2$$

$$- [2C_2(R)T_2(R) + 2C_2(G)T_2(R)]$$

$$= 2C_2(G)T_2(R) + 4C_2(R)T_2(R) - 6C_2(G)^2 \ . \tag{B.1.1.14}$$

B.1.2 Fierzing

A matrix $\mathbf{M}$ can be expressed in terms of the group G's generators

$$\mathbf{M} = f^A \mathbf{T}^A + f^0 \ \mathbf{I} \ . \tag{B.1.2.1}$$

Taking the trace of both sides yields

$$\text{Tr}\{\mathbf{M}\} = f^0 d(R) \ . \tag{B.1.2.2}$$

Multiplying by $\mathbf{T}^B$ and taking the trace gives

$$\text{Tr}\{\mathbf{M}\mathbf{T}^B\} = f^A T_2(R)\delta^{AB} \ . \tag{B.1.2.3}$$

Hence

$$\mathbf{M} = \frac{1}{T_2(R)}\mathrm{Tr}\{\mathbf{MT}^A\}\mathbf{T}^A + \frac{1}{d(R)}\mathrm{Tr}\{\mathbf{M}\}\,\mathbf{I}\,. \qquad \text{(B.1.2.4)}$$

If $\mathbf{M} = \Phi\Phi^\dagger$, it follows that

$$\Phi\Phi^\dagger = \frac{1}{T_2(R)}(\Phi^\dagger\mathbf{T}^A\Phi)\mathbf{T}^A + \frac{1}{d(R)}(\Phi^\dagger\Phi)\,\mathbf{I}\,, \qquad \text{(B.1.2.5)}$$

where use has been made of $\mathrm{Tr}\{\Phi\Phi^\dagger\mathbf{T}^A\} = \mathrm{Tr}\{\Phi^\dagger\mathbf{T}^A\Phi\} = \Phi^\dagger\mathbf{T}^A\Phi$. Now multiplying on the left by $\Phi^\dagger$ and on the right by Φ gives

$$(\Phi^\dagger\Phi)^2 = \frac{1}{T_2(R)}(\Phi^\dagger\mathbf{T}^A\Phi)(\Phi^\dagger\mathbf{T}^A\Phi) + \frac{1}{d(R)}(\Phi^\dagger\Phi)^2\,. \qquad \text{(B.1.2.6)}$$

Combining like terms yields

$$(\Phi^\dagger\mathbf{T}^A\Phi)(\Phi^\dagger\mathbf{T}^A\Phi) = T_2(R)[1 - \frac{1}{d(R)}](\Phi^\dagger\Phi)^2\,. \qquad \text{(B.1.2.7)}$$

In the fundamental representation of $SU(n)$, $T_2(R) = 1/2$ and $d(R) = n$. Taking $SU(2)$ as an example, with Φ a doublet, it follows that

$$(\Phi^\dagger\mathbf{T}^A\Phi)(\Phi^\dagger\mathbf{T}^A\Phi) = \frac{1}{4}(\Phi^\dagger\Phi)^2\,. \qquad \text{(B.1.2.8)}$$

It is also useful to have the analogous expression for the case when there are two different $SU(2)$ fields involved

$$(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) = \frac{1}{T_2(R)}(\Phi_1^\dagger\mathbf{T}^A\Phi_1)(\Phi_2^\dagger\mathbf{T}^A\Phi_2) + \frac{1}{d(R)}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\,,$$
$$\text{(B.1.2.9)}$$

which follows immediately from Eq. (B.1.2.5).

B.2 Anomalous Dimension of the Scalar Field

Using the Machacek-Vaughn convention, the Yukawa sector of the SUSY Standard Model is written

$$\mathcal{L} = \overline{u}_L \mathbf{U} \Phi_u Q_L + \tilde{\overline{u}}_L \mathbf{U} \tilde{\Phi}_u Q_L + \overline{u}_L \mathbf{U} \tilde{\Phi}_u \tilde{Q}_L +$$

$$\overline{d}_L \mathbf{D} \Phi_d Q_L + \tilde{\overline{d}}_L \mathbf{D} \tilde{\Phi}_d Q_L + \overline{d}_L \mathbf{D} \tilde{\Phi}_d \tilde{Q}_L +$$

$$\overline{e}_L \mathbf{E} \Phi_d L_L + \tilde{\overline{e}}_L \mathbf{E} \tilde{\Phi}_d L_L + \overline{e}_L \mathbf{E} \tilde{\Phi}_d \tilde{L}_L + c.c.$$

$$= \overline{u}_{Li\alpha} \mathbf{U}_{ij} \Phi_{uA} (i\sigma_2)_{AB} Q_{Lj\beta B} \delta_{\alpha\beta} + \Phi_{uA}^* (i\sigma_2)_{AB} Q_{Lj\beta B}^* \mathbf{U}_{ji}^\dagger \overline{u}_{Li\alpha}^* + \cdots .$$
$$(B.2.1)$$

The $i\tau_2$'s involved in the Weyl spinor product have been suppressed and so have the $i\sigma_2$'s involved in the $SU(2)$ product, although the $i\sigma_2$'s are displayed in the second step. The $SU(3)$ product follows since the barred fields are anti-quarks. All fermions are represented by Grassmann fields. The $c.c.$ is explicitly calculated in the example below

$$[\chi_L(i\tau_2)\eta_L]^* = \chi_L^*(i\tau_2)^* \eta_L^*$$

$$= \chi_L^*(i\tau_2)\eta_L^*$$

$$= [\overline{\chi}_R(i\tau_2)](i\tau_2)[(-i\tau_2)\overline{\eta}_R]$$
$$(B.2.2)$$

$$= \overline{\chi}_R(i\tau_2)\overline{\eta}_R = \overline{\eta}_R(i\tau_2)\overline{\chi}_R ,$$

where the identity, $\overline{\psi}_R = i\tau_2\psi_L^*$, has been used. The following identifications can be made from Eq.(B.2.1)

$$Y^{+u(\phi A)}_{[(ui\alpha)\bullet](Qj\beta\overline{B})} +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{U}_{ij} , \qquad \dot{Y}^{-u(\phi A)}_{(Qj\beta B)[\bullet(ui\overline{\alpha})]} -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{U}_{ji}^\dagger ,$$

$$Y^{+u(ui\alpha)}_{[\bullet(\phi A)](Qj\beta\overline{B})} +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{U}_{ij} , \qquad \dot{Y}^{-u(ui\alpha)}_{(Qj\beta B)[(\phi A)\overline{\bullet}]} -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{U}_{ji}^\dagger ,$$

$$Y^{+u(Qj\beta B)}_{[(ui\alpha)(\phi A)]\bullet} = +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{U}_{ij} , \qquad \dot{Y}^{-u(Qj\beta B)}_{\bullet[(\phi A)(ui\alpha)]} = -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{U}_{ji}^\dagger ,$$
$$(B.2.3)$$

$$Y^{+d(\phi A)}_{[(di\alpha)\bullet](Qj\beta\overline{B})} +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{D}_{ij} , \qquad \dot{Y}^{-d(\phi A)}_{(Qj\beta B)[\bullet(di\overline{\alpha})]} -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{D}_{ji}^\dagger ,$$

$$Y^{+d(di\alpha)}_{[\bullet(\phi A)](Qj\beta\overline{B})} +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{D}_{ij} , \qquad \dot{Y}^{-d(di\alpha)}_{(Qj\beta B)[(\phi A)\overline{\bullet}]} -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{D}_{ji}^\dagger ,$$

$$Y^{+d(Qj\beta B)}_{[(di\alpha)(\phi A)]\bullet} = +\delta_{\alpha\beta}(i\sigma_2)_{AB}\mathbf{D}_{ij} , \qquad \dot{Y}^{-d(Qj\beta B)}_{\bullet[(\phi A)(di\alpha)]} = -\delta_{\beta\alpha}(i\sigma_2)_{BA}\mathbf{D}_{ji}^\dagger ,$$
$$(B.2.4)$$

$$\mathbf{Y}^{+d(\phi A)}_{[(ei)\bullet](LjB)} = +(i\sigma_2)_{AB}\mathbf{E}_{ij} \, , \qquad \dot{\mathbf{Y}}^{-d(\phi A)}_{(LjB)[\bullet(ei)]} = -(i\sigma_2)_{BA}\mathbf{E}^\dagger_{ji} \, ,$$

$$\mathbf{Y}^{+d(ei)}_{[\bullet(\phi A)](LjB)} = +(i\sigma_2)_{AB}\mathbf{E}_{ij} \, , \qquad \dot{\mathbf{Y}}^{-d(ei)}_{(LjB)[(\phi A)\bullet]} = -(i\sigma_2)_{BA}\mathbf{E}^\dagger_{ji} \, ,$$

$$\mathbf{Y}^{+d(LjB)}_{[(ei)(\phi A)]\bullet} = +(i\sigma_2)_{AB}\mathbf{E}_{ij} \, , \qquad \dot{\mathbf{Y}}^{-d(LjB)}_{\bullet[(\phi A)(ei)]} = -(i\sigma_2)_{BA}\mathbf{E}^\dagger_{ji} \, ,$$

$$\text{(B.2.5)}$$

The $+$ and $-$ on the $\mathbf{Y}$'s denote whether or not the Φ is complex conjugated, respectively. The upper index always denotes the scalar, and a bullet is placed in its unoccupied lower position. Since two of the three fields are either transposed or not (in the spinor, $SU(2)$ doublet, $SU(3)$ triplet, or generation sense), they appear grouped in square brackets. Dots over $\mathbf{Y}$'s indicate that the Yukawa coupling matrix it represents is daggered.

Consider the following example from Ref. 7

$$\chi = \frac{3}{2}\mathrm{Tr}\{\mathbf{Y}^b\mathbf{Y}^{\dagger a}\mathbf{Y}^c\mathbf{Y}^{\dagger c}\} + \mathrm{Tr}\{\mathbf{Y}^b\mathbf{Y}^{\dagger c}\mathbf{Y}^a\mathbf{Y}^{\dagger c}\} \, , \qquad \text{(B.2.6)}$$

a quantity that enters the scalar 2-loop anomalous dimension. In the above notation (suppressing some indices at first), the first term reads (in the Φ_u case)

$$X_1^{(+A)(-B)} = \mathrm{Tr}\{\mathbf{Y}^{+u(\phi A)}\dot{\mathbf{Y}}^{-u(\phi B)}\underline{\mathbf{Y}}\,\underline{\dot{\mathbf{Y}}}\} \, , \qquad \text{(B.2.7)}$$

but to this must be added another term

$$X_2^{(+A)(-B)} = \mathrm{Tr}\{\dot{\mathbf{Y}}^{+u(\phi A)}\mathbf{Y}^{-u(\phi B)}\underline{\dot{\mathbf{Y}}}\,\underline{\mathbf{Y}}\} \, , \qquad \text{(B.2.8)}$$

since there must be alternating daggered $\mathbf{Y}$'s in these expressions, and both equations (B.2.7) and (B.2.8) above satisfy this (the underlines on two $\mathbf{Y}$'s indicate that their scalar indices are to be contracted). Furthermore, the result must be symmetric, in this case, in A and B, so the next two terms must also be included

$$X_1^{(-A)(+B)} = \mathrm{Tr}\{\mathbf{Y}^{-u(\phi A)}\dot{\mathbf{Y}}^{+u(\phi B)}\underline{\mathbf{Y}}\,\underline{\dot{\mathbf{Y}}}\}$$

$$X_2^{(-A)(+B)} = \mathrm{Tr}\{\dot{\mathbf{Y}}^{-u(\phi A)}\mathbf{Y}^{+u(\phi B)}\underline{\dot{\mathbf{Y}}}\,\underline{\mathbf{Y}}\} \, . \qquad \text{(B.2.9)}$$

Putting it all together, the first term in Eq. (B.2.6) is given by

$$X = \frac{1}{2}[(X_1^{+-} + X_2^{+-}) + (X_1^{-+} + X_2^{-+})] . \qquad (B.2.10)$$

Similarly the second term in Eq. (B.2.6) is

$$Y = \frac{1}{2}[(Y_1 + Y_2)^{+-} + (Y_1 + Y_2)^{-+}] . \qquad (B.2.11)$$

Now to compute these contributions (the flavor and color indices are suppressed since these contract in a straightforward way)

$$
\begin{aligned}
X_1^{+-} &= \mathrm{Tr}\{\mathbf{Y}_{u\bullet(QC)}^{+u(\phi A)}\dot{\mathbf{Y}}_{(QC)\bullet u}^{-u(\phi B)}[\mathbf{Y}_{u\bullet(QD)}^{+u(\phi E)}\dot{\mathbf{Y}}_{(QD)\bullet u}^{-u(\phi E)} + \mathbf{Y}_{u(\phi D)\bullet}^{+u(QE)}\dot{\mathbf{Y}}_{\bullet(\phi D)u}^{-u(QE)}]\} \\
&= 3\mathrm{Tr}\{(\mathbf{U}^\dagger\mathbf{U})^2\}(i\sigma_2)_{AC}(-i\sigma_2)_{CB}[(i\sigma_2)_{ED}(-i\sigma_2)_{DE} + (i\sigma_2)_{DE}(-i\sigma_2)_{ED}] \\
&= 12\mathrm{Tr}\{(\mathbf{U}^\dagger\mathbf{U})^2\}\delta_{AB} ,
\end{aligned}
$$

$$X_2^{+-} = 0 ,$$

$$X_1^{-+} = 0 ,$$

$$
\begin{aligned}
X_2^{-+} &= \mathrm{Tr}\{\dot{\mathbf{Y}}_{(QC)\bullet u}^{-u(\phi A)}\mathbf{Y}_{u\bullet(QD)}^{+u(\phi B)}[\dot{\mathbf{Y}}_{(QD)\bullet u}^{-u(\phi E)}\mathbf{Y}_{u\bullet(QC)}^{+u(\phi E)} + \dot{\mathbf{Y}}_{(QD)\bullet d}^{-d(\phi E)}\mathbf{Y}_{d\bullet(QC)}^{+d(\phi E)} \\
&\qquad + \dot{\mathbf{Y}}_{(QD)(\phi E)\bullet}^{-u(u)}\mathbf{Y}_{\bullet(\phi E)(QC)}^{+u(u)} + \dot{\mathbf{Y}}_{(QD)(\phi E)\bullet}^{-d(d)}\mathbf{Y}_{\bullet(\phi E)(QC)}^{+d(d)}]\} \\
&= 2\cdot 3\mathrm{Tr}\{(\mathbf{U}^\dagger\mathbf{U})^2\mathbf{U}^\dagger\mathbf{U}\mathbf{D}^\dagger\mathbf{D}\}(-i\sigma_2)_{CA}(i\sigma_2)_{BD}(-i\sigma_2)_{DE}(i\sigma_2)_{EC} \\
&= 6\mathrm{Tr}\{(\mathbf{U}^\dagger\mathbf{U})^2 + \mathbf{U}^\dagger\mathbf{U}\mathbf{D}^\dagger\mathbf{D}\}\delta_{AB} .
\end{aligned}
$$

$$(B.2.12)$$

Some of the 3's come from the trace over the suppressed color delta functions. Finally,

$$X = 3\mathrm{Tr}\{3(\mathbf{U}^\dagger\mathbf{U})^2 + \mathbf{U}^\dagger\mathbf{U}\mathbf{D}^\dagger\mathbf{D}\}\delta_{AB} . \qquad (B.2.13)$$

Similarly,

$$
\begin{aligned}
Y_1^{+-} &= \mathrm{Tr}\{\mathbf{Y}^{+u(\phi A)}\underline{\dot{\mathbf{Y}}}\mathbf{Y}^{-u(\phi B)}\underline{\dot{\mathbf{Y}}}\} = 0 , \\
Y_2^{+-} &= \mathrm{Tr}\{\dot{\mathbf{Y}}^{+u(\phi A)}\underline{\mathbf{Y}}\dot{\mathbf{Y}}^{-u(\phi B)}\underline{\mathbf{Y}}\} = 0 , \\
Y_1^{-+} &= \mathrm{Tr}\{\mathbf{Y}^{-u(\phi A)}\underline{\dot{\mathbf{Y}}}\mathbf{Y}^{+u(\phi B)}\underline{\dot{\mathbf{Y}}}\} = 0 , \\
Y_2^{-+} &= \mathrm{Tr}\{\dot{\mathbf{Y}}^{-u(\phi A)}\underline{\mathbf{Y}}\dot{\mathbf{Y}}^{+u(\phi B)}\underline{\mathbf{Y}}\} = 0 .
\end{aligned}
$$

$$(B.2.14)$$

The full answer is then

$$\chi u = \frac{3}{2}X + Y = \frac{9}{2}\text{Tr}\{3(\mathbf{U}^\dagger\mathbf{U})^2 + \mathbf{U}^\dagger\mathbf{U}\mathbf{D}^\dagger\mathbf{D}\} \ . \qquad (B.2.15)$$

B.3 Yukawa Couplings

The Yukawa interactions arising from the D-terms in the SUSY Lagrangian can be written

$$\mathcal{L}^{SUSY}_{Yukawa} = -i\sqrt{2}g[\phi_i^* \ T^R_{ij} \ \psi_j\lambda^R - \overline{\psi}_j \ T^R_{ji} \ \phi_i\overline{\lambda}^R] \ , \qquad (B.3.1)$$

where the overlined fermi (Grassmann) fields are right-handed. Now make the correspondence

$$Y^a_{ij}\xrightarrow{\ SUSY\ }\tilde{Y}^{+(\lambda R)}_{(\square_i)(\square_j)} = -i\sqrt{2}gT^R_{ij} \ ,$$
$$\tilde{Y}^{-(\lambda R)}_{(\square_j)(\square_i)} = +i\sqrt{2}gT^R_{ji} \ . \qquad (B.3.2)$$

The $\square$'s $\triangle$'s, $\odot$'s, etc. represent field-types (i.e., Q, $\overline{u}$, $\overline{d}$, etc.). Note that the $\tilde{Y}$'s which represent T^R's are diagonal in type (i.e., both indices are $\square$'s as exemplified above). The $+$ and $-$ indicate that the scalar index is complex conjugated (equivalently, is the left subscript on $\tilde{Y}$'s) or not complex conjugated (equivalently, is the right subscript on $\tilde{Y}$'s), respectively. Written this way, the i above represents the ith group component of the scalar, and the j represents the group component of the fermion. Contracting two $\tilde{Y}$'s (i.e., using underscores) implies in their case contracting the superscript, group generator indices (the R's).

In order to construct the one loop SUSY beta function formula for the Yukawa matrices the following must be supersymmetrized

$$\underline{YY}^{\dagger}Y^{a} \xrightarrow{SUSY} \left(\tilde{Y}^{\pm(\lambda R)}\tilde{Y}^{\mp(\lambda R)}Y^{+(\phi)}\right)_{[\Box\bullet]_{\triangle}}$$

$$= \tilde{Y}^{-(\lambda R)}_{\Box\Box}\tilde{Y}^{+(\lambda R)}_{\Box\Box}Y^{+(\phi)}_{[\Box\bullet]_{\triangle}} + \tilde{Y}^{+(\lambda R)}_{\phi\phi}\tilde{Y}^{-(\lambda R)}_{\phi\phi}Y^{+(\phi)}_{[\Box\bullet]_{\triangle}}$$

$$= \left(C_{2}(\Box) + C_{2}(\phi)\right)Y^{+(\phi)}_{[\Box\bullet]_{\triangle}}$$

$$\rightarrow 2g^{2}C_{2}(R)Y^{(\phi)} \ ,$$

$$Y^{a}\underline{Y}^{\dagger}\underline{Y} \xrightarrow{SUSY} \left(Y^{+(\phi)}\tilde{Y}^{\pm(\lambda R)}\tilde{Y}^{\mp(\lambda R)}\right)_{[\Box\bullet]_{\triangle}}$$

$$= Y^{+(\phi)}_{[\Box\bullet]_{\triangle}}\tilde{Y}^{-(\lambda R)}_{\triangle\triangle}\tilde{Y}^{+(\lambda R)}_{\triangle\triangle}$$

$$= Y^{+(\phi)}_{[\Box\bullet]_{\triangle}}C_{2}(\triangle)$$

$$\rightarrow 2g^{2}Y^{(\phi)}C_{2}(R) \ .$$

(B.3.3)

Furthermore, in obtaining the formulas for the beta functions of the different Yukawas the "external subscripts" are generally two fermions (not scalarinos) which are represent by $\Box$'s, $\triangle$'s, $\odot$'s, etc., and the "external superscript" is a scalar (a Higgs not an sfermion) which is represented by a (ϕ). Hence, it should be clear from inspection that the following terms cannot be constructed for any choice of "internal subscripts"

$$\underline{Y}\text{Tr}\{\underline{Y}^{\dagger}Y^{a}\} \xrightarrow{SUSY} (\tilde{Y}^{\pm(\lambda R)}\text{Tr}\{\tilde{Y}^{\mp(\lambda R)}Y^{(\phi)}\})_{\Box\triangle} \rightarrow 0 \ ,$$

$$\underline{Y}Y^{a\dagger}\underline{Y} \xrightarrow{SUSY} (\tilde{Y}^{\pm(\lambda R)}Y^{(\phi)}\tilde{Y}^{\mp(\lambda R)})_{\Box\triangle} \rightarrow 0 \ .$$

(B.3.4)

Application of Eq. (B.3.3) to the up-Yukawa ($Y^{+u(\phi)}_{[u\bullet](QB)}$; see section where the SUSY Y's are defined) yields

$$\left(C_{2}(R)Y\right)^{+u(\phi)}_{[u\bullet](QB)} = C_{2u\Box}Y^{+u(\phi)}_{[\Box\bullet](QB)} + C_{2\Phi_{u}\Box}Y^{\Box}_{[u\bullet](QB)}$$

$$= \left(C_{2}(\overline{u}) + C_{2}(\Phi_{U})\right)Y^{+u(\phi)}_{[u\bullet](QB)} \ ,$$

(B.3.5)

where use was made of the diagonal nature of $C_{2\square\triangle} \sim \delta_{\square\triangle}C_2(\square)$. Note that C_2 on the left can act on both left subscripts. Likewise

$$\left(\mathbf{Y}C_2(R)\right)^{+u(\phi)}_{[u\bullet](QB)} = \mathbf{Y}^{+u(\phi)}_{[u\bullet]\square}C_{2\square(QB)}$$

$$= C_2(Q)\mathbf{Y}^{+u(\phi)}_{[u\bullet](QB)} \ . \tag{B.3.6}$$

The standard one loop beta function for the Yukawas is

$$(4\pi)^2\beta^a = \frac{1}{2}\left[\underline{\mathbf{Y}\mathbf{Y}}^\dagger\mathbf{Y}^a + \mathbf{Y}^a\underline{\mathbf{Y}^\dagger\mathbf{Y}}\right] + 2\underline{\mathbf{Y}}\mathbf{Y}^{a\dagger}\underline{\mathbf{Y}} + \underline{\mathbf{Y}}\text{Tr}\{\underline{\mathbf{Y}}^\dagger\mathbf{Y}^a\}$$

$$- 3g^2\Big\{\mathbf{C}_2(F), \mathbf{Y}^a\Big\} \ . \tag{B.3.7}$$

Supersymmetrizing according to the results derived above gives

$$(4\pi)^2\beta^a_{SUSY} = \frac{1}{2}\left[\underline{\mathbf{Y}\mathbf{Y}}^\dagger\mathbf{Y}^a + \mathbf{Y}^a\underline{\mathbf{Y}^\dagger\mathbf{Y}}\right] + 2\underline{\mathbf{Y}}\mathbf{Y}^{a\dagger}\underline{\mathbf{Y}} + \underline{\mathbf{Y}}\text{Tr}\{\underline{\mathbf{Y}}^\dagger\mathbf{Y}^a\}$$

$$- (3 - \tilde{\delta}_g)g^2\Big\{\mathbf{C}_2(R), \mathbf{Y}^a\Big\} \ , \tag{B.3.8}$$

where $\tilde{\delta}_g$ equals 1 or 0 depending on whether one is above or below, respectively, the mass threshold of the gaugino in question. The labels T_i, $i = 1,\ldots,5$, will be used to denote the five terms appearing in Eq. (B.3.8) when computing the supersymmetric beta function for the up-Yukawa

$$T_1 = \left(\underline{\mathbf{Y}\mathbf{Y}}^\dagger\mathbf{Y}^{+u(\phi)}\right)_{[u\bullet](QB)}$$

$$= \left[\mathbf{Y}^{+u(\phi D)}_{[u\bullet](QC)}\dot{\mathbf{Y}}^{-u(\phi D)}_{(QC)[\bullet u]} + \mathbf{Y}^{+u(QD)}_{[u(\phi C)]\bullet}\dot{\mathbf{Y}}^{-u(QD)}_{\bullet[(\phi C)u]}\right]\mathbf{Y}^{+u(\phi A)}_{[u\bullet](QB)}$$

$$= \left[(i\sigma_2)_{DC}(-i\sigma_2)_{CD} + (i\sigma_2)_{CD}(-i\sigma_2)_{DC}\right](i\sigma_2)_{AB}\mathbf{U}\mathbf{U}^\dagger\mathbf{U}$$

$$= (i\sigma_2)_{AB}\mathbf{U}\left[4\mathbf{U}^\dagger\mathbf{U}\right] \ , \tag{B.3.9}$$

$$T_2 = \left(\mathbf{Y}^{+u(\phi)}\underline{\mathbf{Y}^\dagger\mathbf{Y}}\right)_{[u\bullet](QB)}$$

$$= \mathbf{Y}^{+u(\phi A)}_{[u\bullet](QC)}\left[\dot{\mathbf{Y}}^{-u(\phi D)}_{(QC)[\bullet u]}\mathbf{Y}^{+u(\phi D)}_{[u\bullet](QB)} + \dot{\mathbf{Y}}^{-u(u)}_{(QC)[(\phi D)\bullet]}\mathbf{Y}^{+u(u)}_{[\bullet(\phi D)](QB)}\right.$$

$$\left.+ \dot{\mathbf{Y}}^{-d(\phi D)}_{(QC)[\bullet d]}\mathbf{Y}^{+d(\phi D)}_{[d\bullet](QB)} + \dot{\mathbf{Y}}^{-d(d)}_{(QC)[(\phi D)\bullet]}\mathbf{Y}^{+d(d)}_{[\bullet(\phi D)](QB)}\right]$$

$$= (i\sigma_2)_{AC}\mathbf{U}\left[(-i\sigma_2)_{CD}(i\sigma_2)_{DB}\mathbf{U}^\dagger\mathbf{U} + (-i\sigma_2)_{CD}(i\sigma_2)_{DB}\mathbf{U}^\dagger\mathbf{U}\right.$$

$$\left.+ (-i\sigma_2)_{CD}(i\sigma_2)_{DB}\mathbf{D}^\dagger\mathbf{D} + (-i\sigma_2)_{CD}(i\sigma_2)_{DB}\mathbf{D}^\dagger\mathbf{D}\right]$$

$$= (i\sigma_2)_{AB}\mathbf{U}\left[2\mathbf{U}^\dagger\mathbf{U} + 2\mathbf{D}^\dagger\mathbf{D}\right] \ , \tag{B.3.10}$$

$$T_3 = \left(\underline{\mathbf{Y}}\dot{\mathbf{Y}}^{+u(\phi)}\underline{\mathbf{Y}}\right)_{[u\bullet](QB)} \tag{B.3.11}$$

$$= 0 \;,$$

$$T_4 = \left(\underline{\mathbf{Y}}\mathrm{Tr}\{\underline{\mathbf{Y}}^\dagger\mathbf{Y}^{+u(\phi)}\}\right)_{[u\bullet](QB)}$$

$$= \mathbf{Y}^{+u(\phi C)}_{[u\bullet](QB)}\mathrm{Tr}\{\dot{\mathbf{Y}}^{-u(\phi C)}_{(QD)[\bullet u]}\mathbf{Y}^{+u(\phi A)}_{[u\bullet](QD)}\}$$

$$= (i\sigma_2)_{CB}\mathbf{U}\mathrm{Tr}\{3\mathbf{U}^\dagger\mathbf{U}\}(-i\sigma_2)_{DC}(i\sigma_2)_{AD} \tag{B.3.12}$$

$$= (i\sigma_2)_{AB}\mathbf{U}\left[\mathrm{Tr}\{3\mathbf{U}^\dagger\mathbf{U}\}\right]\;,$$

$$T_5 = \left(\left\{\mathbf{C}_2(R), \mathbf{Y}^{+u(\phi)}\right\}\right)_{[u\bullet](QB)}$$

$$= (i\sigma_2)_{AB}\mathbf{U}\sum_{k=1}^{3}\left[\mathbf{C}_2{}^k(\overline{u}) + \mathbf{C}_2{}^k(Q) + \mathbf{C}_2{}^k(\Phi_u)\right]$$

$$= (i\sigma_2)_{AB}\mathbf{U}\left[(\frac{4}{9} + \frac{1}{36} + \frac{1}{4})g^2 + (0 + \frac{3}{4} + \frac{3}{4})g_2^2 \right. \tag{B.3.13}$$

$$\left. + (\frac{4}{3} + \frac{4}{3} + 0)g_3^2\right]$$

$$= (i\sigma_2)_{AB}\mathbf{U}\left[\frac{13}{18}g^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right]\;.$$

In T_4 the 3 comes from the trace over the suppressed color indices. Also, in T_5 use was made of the fact that if the gauge group is not simple but a direct product $G_1 \times \cdots \times G_n$ with couplings $g_1, \ldots, g_n$, then $g^2\mathbf{C}_2 \to \sum_{k=1}^{n} g_k^2\mathbf{C}_2{}^k$.

B.4 Thresholds

To implement the super particle thresholds in the minimal low energy super gravity model, the renormalization group β function must be calculated in a form that has not appeared in the literature. In the following example, the one loop β function of g_3 is considered. Starting from Eq. (B.1.1.12)

$$(4\pi)^2\beta_3^{(1)}/g_3^3 = \frac{2}{3}T_2(F_3) + \frac{1}{3}T_2(S_3) + \frac{2}{3}C_2(G_3)\tilde{\delta}_g - \frac{11}{3}C_2(G_3)\;, \tag{B.4.1}$$

where F_3, S_3 refer to the fermion and scalar representations, and $G_3 = SU(3)$. Also

$$\tilde{\delta}_g = \begin{cases} 1\;, & \mu > M_g \;; \\ 0\;, & \mu < M_g \;, \end{cases} \tag{B.4.2}$$

where M_g is the mass of the gluino. When dealing with a direct product group, like $SU(3) \times SU(2) \times U(1)$,

$$T_2(R_3) \rightarrow T_2(R_3)d(R_1)d(R_2) . \tag{B.4.3}$$

From the definition of $T_2(R)$ in Eq. (B.1.1.2), one obtains the following result in the $SU(3)$ case

$$T_2(R_3) = 2(\frac{1}{2})N_Q + (\frac{1}{2})N_{\bar{u}} + (\frac{1}{2})N_{\bar{d}} , \tag{B.4.4}$$

where N_p equals the number of generations of particle p. This result is valid for both fermion $(R = F)$ and scalar $(R = S)$ representations. The notation will be such that $N = n$ for the particle and $N = \tilde{n}$ for the SUSY partner. Equations (B.4.1) and (B.4.4) lead to

$$(4\pi)^2 \beta_3^{(1)}/g_3^3 = \frac{2}{3}(n_u + n_d) + \frac{1}{3}\tilde{n}_Q + \frac{1}{6}\tilde{n}_{\bar{u}} + \frac{1}{6}\tilde{n}_{\bar{d}} + 2\tilde{\delta}_g - 11 . \tag{B.4.5}$$

It has been assumed that $n_Q = (n_u + n_d)/2$. Also the fact that left-handed and right-handed quarks of a given flavor have the same mass implies $n_u = n_{\bar{u}}$ and $n_d = n_{\bar{d}}$. Note that this reduces to the right standard model result when $\tilde{n} = \tilde{\delta}_g = 0$, and to the right supersymmetric result $(\mu > M_{SUSY})$ when $\tilde{n} = 3$, $\tilde{\delta}_g = 1$. Similar formulas are calculated for g_1 and g_2

$$(4\pi)^2 \beta_1^{(1)}/g_1^3 = \frac{2}{5}(\frac{17}{12}n_u + \frac{5}{12}n_d + \frac{5}{4}n_e + \frac{1}{4}n_\nu) + \frac{1}{30}\tilde{n}_Q + +\frac{4}{15}\tilde{n}_{\bar{u}} + \frac{1}{15}\tilde{n}_{\bar{d}}$$
$$+ \frac{1}{10}\tilde{n}_L + \frac{1}{5}\tilde{n}_{\bar{e}} + \frac{1}{5}(\tilde{n}_{\phi_u} + \tilde{n}_{\phi_d}) + \frac{1}{10}(n_{\phi_u} + n_{\phi_d}) ,$$
$$(4\pi)^2 \beta_2^{(1)}/g_2^3 = -\frac{22}{3} + \frac{1}{2}(n_u + n_d) + \frac{1}{6}(n_e + n_\nu) + \frac{1}{2}\tilde{n}_Q + \frac{1}{6}\tilde{n}_L$$
$$+ \frac{1}{3}(\tilde{n}_{\phi_u} + \tilde{n}_{\phi_d}) + \frac{1}{6}(n_{\phi_u} + n_{\phi_d}) + \frac{4}{3}\tilde{\delta}_W .$$
$$\tag{B.4.6}$$

For the gauge couplings, the two loop contributions were also calculated in this manner. For the Yukawa couplings, this form of the β function was calculated to one loop.

	Table 6: Group Theory Factors								
	$T_2(R)$			$C_2(R)$			$d(R)$		
	U_1	SU_2	SU_3	U_1	SU_2	SU_3	U_1	SU_2	SU_3
Q	$(+\frac{1}{6})^2$	$\frac{1}{2}$	$\frac{1}{2}$	$(+\frac{1}{6})^2$	$\frac{3}{4}$	$\frac{4}{3}$	1	2	3
$\overline{u}$	$(-\frac{2}{3})^2$	0	$\frac{1}{2}$	$(-\frac{2}{3})^2$	0	$\frac{4}{3}$	1	1	3
$\overline{d}$	$(+\frac{1}{3})^2$	0	$\frac{1}{2}$	$(+\frac{1}{3})^2$	0	$\frac{4}{3}$	1	1	3
L	$(-\frac{1}{2})^2$	$\frac{1}{2}$	0	$(-\frac{1}{2})^2$	$\frac{3}{4}$	0	1	2	1
$\overline{e}$	$(+1)^2$	0	0	$(+1)^2$	0	0	1	1	1
Φ_u	$(+\frac{1}{2})^2$	$\frac{1}{2}$	0	$(+\frac{1}{2})^2$	$\frac{3}{4}$	0	1	2	1
Φ_d	$(-\frac{1}{2})^2$	$\frac{1}{2}$	0	$(-\frac{1}{2})^2$	$\frac{3}{4}$	0	1	2	1

Table 7		
G	$C_2(G)$	$d(G)$
U_1	0	1
SU_2	2	3
SU_3	3	8

APPENDIX C
THE MSSM β FUNCTIONS

Using some of the notation of Falck [80], the superpotential and soft symmetry breaking potential are as follows

$$W = \hat{\tilde{u}}\mathbf{Y}_u \hat{\Phi}_u \hat{Q} + \hat{\tilde{d}}\mathbf{Y}_d \hat{\Phi}_d \hat{Q} + \hat{\tilde{e}}\mathbf{Y}_e \Phi_d \hat{L} + \breve{\mu}\hat{\Phi}_u \hat{\Phi}_d + h.c. \, ,$$

$$V_{soft} = m_{\phi_u}^2 \Phi_u^\dagger \Phi_u + m_{\phi_d}^2 \Phi_d^\dagger \Phi_d + B\breve{\mu}(\Phi_u \Phi_d + h.c.)$$

$$+ m_Q^2 \tilde{Q}^\dagger \tilde{Q} + m_L^2 \tilde{L}^\dagger \tilde{L} + m_u^2 \tilde{\tilde{u}}^\dagger \tilde{\tilde{u}} + m_d^2 \tilde{\tilde{d}}^\dagger \tilde{\tilde{d}} + m_e^2 \tilde{\tilde{e}}^\dagger \tilde{\tilde{e}}$$

$$+ \sum_{i,j} (A_u^{ij} Y_u^{ij} \tilde{\tilde{u}}_i \Phi_u \tilde{Q}_j + A_d^{ij} Y_d^{ij} \tilde{\tilde{d}}_i \Phi_d \tilde{Q}_j + A_e^{ij} Y_e^{ij} \tilde{\tilde{e}}_i \Phi_d \tilde{L}_j + h.c. \,) \, ,$$

$$V_{gaugino} = \frac{1}{2}\sum_{l=1}^3 M_l \lambda_l \lambda_l + h.c. \, .$$

$$(C.1)$$

Various σ_2's have been omitted, and a sum over the number of generations is implied in the squark and slepton mass terms. Also, hats imply superfields and tildes the superpartners of the given fields.

First the gauge couplings

$$\frac{dg_l}{dt} = -\frac{1}{16\pi^2}b_l g_l^3$$

$$- \frac{g_l^3}{(16\pi^2)^2}\left[\sum_k b_{lk} g_k^2 - \mathrm{Tr}\{C_{lu}\mathbf{Y}_u{}^\dagger \mathbf{Y}_u + C_{ld}\mathbf{Y}_d{}^\dagger \mathbf{Y}_d + C_{le}\mathbf{Y}_e{}^\dagger \mathbf{Y}_e\}\right]$$

$$(C.2)$$

where $t = \ln\mu$ and $l = 1, 2, 3$, corresponding to gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the Standard Model. The various coefficients are defined

to be

$$b_1 = -\frac{3}{5} - 2n_g \ ,$$

$$b_2 = 5 - 2n_g \ , \tag{C.3}$$

$$b_3 = 9 - 2n_g \ ,$$

$$(b_{lk}) = \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & -17 & 0 \\ 0 & 0 & -54 \end{pmatrix} \ , \tag{C.4}$$

and

$$(C_{lf}) = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \ , \text{ with } f = u \ , d \ , e \ , \tag{C.5}$$

with $n_g = \frac{1}{2}n_{fl}$.

In the following, the beta functions for the parameters of the superpotential are listed.

$$\frac{d\ln\tilde\mu}{dt} = \frac{1}{16\pi^2}[\ \mathrm{Tr}\{3Y_u{}^\dagger Y_u + 3Y_d{}^\dagger Y_d + Y_e{}^\dagger Y_e\} - 3(\frac{1}{5}g_1^2 + g_2^2) \ ] \ . \tag{C.6}$$

In the Yukawa sector the β functions are

$$\frac{dY_{u,d,e}}{dt} = Y_{u,d,e}(\frac{1}{16\pi^2}\beta_{u,d,e}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{u,d,e}^{(2)}) \ , \tag{C.7}$$

where the one-loop contributions are given by

$$\beta_u^{(1)} = 3Y_u{}^\dagger Y_u + Y_d{}^\dagger Y_d + 3\mathrm{Tr}\{Y_u{}^\dagger Y_u\} - (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \ ,$$

$$\beta_d^{(1)} = 3Y_d{}^\dagger Y_d + Y_u{}^\dagger Y_u + \mathrm{Tr}\{3Y_d{}^\dagger Y_d + Y_e{}^\dagger Y_e\} - (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \ ,$$

$$\beta_e^{(1)} = 3Y_e{}^\dagger Y_e + \mathrm{Tr}\{3Y_d{}^\dagger Y_d + Y_e{}^\dagger Y_e\} - (\frac{9}{5}g_1^2 + 3g_2^2) \ ,$$

$$\tag{C.8}$$

and the two-loop contributions are given by

$$
\begin{aligned}
\beta_u^{(2)} = {}& -4(\mathbf{Y}_u{}^\dagger\mathbf{Y}_u)^2 - 2(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d)^2 - 2\mathbf{Y}_d{}^\dagger\mathbf{Y}_d\mathbf{Y}_u{}^\dagger\mathbf{Y}_u - 9\text{Tr}\{\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\}\mathbf{Y}_u{}^\dagger\mathbf{Y}_u \\
& - \text{Tr}\{3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e{}^\dagger\mathbf{Y}_e\}\mathbf{Y}_d{}^\dagger\mathbf{Y}_d - 3\text{Tr}\{3(\mathbf{Y}_u{}^\dagger\mathbf{Y}_u)^2 + \mathbf{Y}_d{}^\dagger\mathbf{Y}_d\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\} \\
& + (\frac{2}{5}g_1^2 + 6g_2^2)\mathbf{Y}_u{}^\dagger\mathbf{Y}_u + (\frac{2}{5}g_1^2)\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + (\frac{4}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\} \\
& + (\frac{26}{15}n_g + \frac{403}{450})g_1^4 + (6n_g - \frac{21}{2})g_2^4 + (\frac{32}{3}n_g - \frac{304}{9})g_3^4 \\
& + g_1^2g_2^2 + \frac{136}{15}g_1^2g_3^2 + 8g_2^3g_3^3 \\[4pt]
\beta_d^{(2)} = {}& -4(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d)^2 - 2(\mathbf{Y}_u{}^\dagger\mathbf{Y}_u)^2 - 2\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\mathbf{Y}_d{}^\dagger\mathbf{Y}_d - 3\text{Tr}\{\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\}\mathbf{Y}_u{}^\dagger\mathbf{Y}_u \\
& - 3\text{Tr}\{3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e{}^\dagger\mathbf{Y}_e\}\mathbf{Y}_d{}^\dagger\mathbf{Y}_d \\
& - 3\text{Tr}\{3(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d)^2 + (\mathbf{Y}_e{}^\dagger\mathbf{Y}_e)^2 + \mathbf{Y}_d{}^\dagger\mathbf{Y}_d\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\} + (\frac{4}{5}g_1^2)\mathbf{Y}_u{}^\dagger\mathbf{Y}_u \\
& + (\frac{4}{5}g_1^2 + 6g_2^2)\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + (-\frac{2}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_d{}^\dagger\mathbf{Y}_d\} \\
& + (\frac{6}{5}g_1^2)\text{Tr}\{\mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} + (\frac{14}{15}n_g + \frac{7}{18})g_1^4 + (6n_g - \frac{21}{2})g_2^4 \\
& + (\frac{32}{3}n_g - \frac{304}{9})g_3^4 + g_1^2g_2^2 + \frac{8}{9}g_1^2g_3^2 + 8g_2^2g_3^2 \\[4pt]
\beta_e^{(2)} = {}& -4(\mathbf{Y}_e{}^\dagger\mathbf{Y}_e)^2 - 3\text{Tr}\{3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e{}^\dagger\mathbf{Y}_e\}\mathbf{Y}_e{}^\dagger\mathbf{Y}_e \\
& - 3\text{Tr}\{3(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d)^2 + (\mathbf{Y}_e{}^\dagger\mathbf{Y}_e)^2 + \mathbf{Y}_d{}^\dagger\mathbf{Y}_d\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\} \\
& + (6g_2^2)\mathbf{Y}_e{}^\dagger\mathbf{Y}_e + (\frac{6}{5}g_1^2)\text{Tr}\{\mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} + (-\frac{2}{5}g_1^2 + 16g_3^2)\text{Tr}\{\mathbf{Y}_d{}^\dagger\mathbf{Y}_d\} \\
& + (\frac{18}{5}n_g + \frac{27}{10})g_1^4 + (6n_g - \frac{21}{2})g_2^4 + \frac{9}{5}g_1^2g_2^2
\end{aligned}
\tag{C.9}
$$

The evolution of the vacuum expectation values of the Higgs's is given by

$$
\frac{d\ln v_{\phi_u,\phi_d}}{dt} = \frac{1}{16\pi^2}\gamma_{\phi_u,\phi_d}^{(1)} + \frac{1}{(16\pi^2)^2}\gamma_{\phi_u,\phi_d}^{(2)} ,
\tag{C.10}
$$

where the one-loop contribution is given by

$$
\begin{aligned}
\gamma_{\phi_u}^{(1)} &= \frac{3}{4}(\frac{1}{5}g_1^2 + g_2^2) - 3\text{Tr}\{\mathbf{Y}_u{}^\dagger\mathbf{Y}_u\} , \\
\gamma_{\phi_d}^{(1)} &= \frac{3}{4}(\frac{1}{5}g_1^2 + g_2^2) - 3\text{Tr}\{\mathbf{Y}_d{}^\dagger\mathbf{Y}_d\} - \text{Tr}\{\mathbf{Y}_e{}^\dagger\mathbf{Y}_e\} ,
\end{aligned}
\tag{C.11}
$$

and the two-loop contribution is given by

$$
\begin{aligned}
\gamma_{\phi_u}^{(2)} &= \frac{3}{4}\mathrm{Tr}\{3(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^2 + 3\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\} \\
&\quad - (\frac{19}{10}g_1^2 + \frac{9}{2}g_2^2 + 20g_3^2)\mathrm{Tr}\{\mathbf{Y}_u^\dagger\mathbf{Y}_u\} \\
&\quad - (\frac{279}{800} + \frac{1803}{1600}n_g)g_1^4 - (\frac{207}{32} + \frac{357}{64}n_g)g_2^4 - (\frac{27}{80} + \frac{9}{80}n_g)g_1^2 g_2^2 \ , \\
\gamma_{\phi_d}^{(2)} &= \frac{3}{4}\mathrm{Tr}\{3(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^2 + 3\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u + (\mathbf{Y}_e^\dagger\mathbf{Y}_e)^2\} \\
&\quad - (\frac{2}{5}g_1^2 + \frac{9}{2}g_2^2 + 20g_3^2)\mathrm{Tr}\{\mathbf{Y}_d^\dagger\mathbf{Y}_d\} - (\frac{9}{5}g_1^2 + \frac{3}{2}g_2^2)\mathrm{Tr}\{\mathbf{Y}_e^\dagger\mathbf{Y}_e\} \\
&\quad - (\frac{279}{800} + \frac{1803}{1600}n_g)g_1^4 - (\frac{207}{32} + \frac{357}{64}n_g)g_2^4 - (\frac{27}{80} + \frac{9}{80}n_g)g_1^2 g_2^2 \ .
\end{aligned}
\tag{C.12}
$$

The renormalization group equations for soft symmetry breaking terms are

$$
\begin{aligned}
\frac{dA_e^{ij}}{dt} &= \frac{1}{16\pi^2}\Big[\, 4(\mathbf{Y}_e\mathbf{Y}_e^\dagger)^{ik}A_e^{kj}\frac{Y_e^{kj}}{Y_e^{ij}} + 5A_e^{ik}\frac{Y_e^{ik}}{Y_e^{ij}}(\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{kj} - 3\frac{A_e^{ij}}{Y_e^{ij}}(\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij} \\
&\quad + 2(A_e^{km}|Y_e^{km}|^2 + 3A_d^{km}|Y_d^{km}|^2) - 6(\frac{3}{5}g_1^2 M_1 + g_2^2 M_2)\,\Big] \ , \\
\frac{dA_d^{ij}}{dt} &= \frac{1}{16\pi^2}\Big[\, 4(\mathbf{Y}_d\mathbf{Y}_d^\dagger)^{ik}A_d^{kj}\frac{Y_d^{kj}}{Y_d^{ij}} + 5A_d^{ik}\frac{Y_d^{ik}}{Y_d^{ij}}(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{kj} - 3\frac{A_d^{ij}}{Y_d^{ij}}(\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij} \\
&\quad + (A_d^{ik} - A_d^{ij})(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{kj}\frac{Y_d^{ik}}{Y_d^{ij}} + 2(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{ik}A_u^{kj}\frac{Y_u^{kj}}{Y_d^{ij}} \\
&\quad + 2(A_e^{km}|Y_e^{km}|^2 + 3A_d^{km}|Y_d^{km}|^2) - \frac{14}{15}g_1^2 M_1 - 6g_2^2 M_2 - \frac{32}{3}g_3^2 M_3 \,\Big] \ , \\
\frac{dA_u^{ij}}{dt} &= \frac{1}{16\pi^2}\Big[\, 4(\mathbf{Y}_u\mathbf{Y}_u^\dagger)^{ik}A_u^{kj}\frac{Y_u^{kj}}{Y_u^{ij}} + 5A_u^{ik}\frac{Y_u^{ik}}{Y_u^{ij}}(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{kj} - 3\frac{A_u^{ij}}{Y_u^{ij}}(\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij} \\
&\quad + (A_u^{ik} - A_u^{ij})(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{kj}\frac{Y_u^{ik}}{Y_u^{ij}} + 2(\mathbf{Y}_u\mathbf{Y}_d^\dagger)^{ik}A_d^{kj}\frac{Y_d^{kj}}{Y_u^{ij}} \\
&\quad + 6A_u^{km}|Y_u^{km}|^2 - \frac{26}{15}g_1^2 M_1 - 6g_2^2 M_2 - \frac{32}{3}g_3^2 M_3 \,\Big] \ ,
\end{aligned}
\tag{C.13}
$$

$$\frac{dm^2_{\phi_u}}{dt} = \frac{1}{8\pi^2}[\sum_{i,j} 3|Y^{ji}_u|^2(m^2_{\phi_u} + m^2_{Q_i} + m^2_{u_j} + |A^{ji}_u|^2)$$

$$+ \frac{3}{10}g^2_1 \text{Tr}\{Ym^2\} - \frac{3}{5}g^2_1 M^2_1 - 3g^2_2 M^2_2\ ],$$

$$\frac{dm^2_{\phi_d}}{dt} = \frac{1}{8\pi^2}[\sum_{i,j}(|Y^{ji}_e|^2(m^2_{\phi_d} + m^2_{L_i} + m^2_{e_j} + |A^{ji}_e|^2)$$

$$+ 3|Y^{ji}_d|^2(m^2_{\phi_d} + m^2_{Q_i} + m^2_{d_j} + |A^{ji}_d|^2))$$

$$- \frac{3}{10}g^2_1 \text{Tr}\{Ym^2\} - \frac{3}{5}g^2_1 M^2_1 - 3g^2_2 M^2_2\ ],$$

$$\frac{dm^2_{e_i}}{dt} = \frac{1}{8\pi^2}[\sum_j 2|Y^{ij}_e|^2(m^2_{\phi_d} + m^2_{e_i} + m^2_{L_j} + |A^{ij}_e|^2)$$

$$+ \frac{3}{5}g^2_1 \text{Tr}\{Ym^2\} - \frac{12}{5}g^2_1 M^2_1\ ],$$

$$\frac{dm^2_{L_i}}{dt} = \frac{1}{8\pi^2}[\sum_j |Y^{ji}_e|^2(m^2_{\phi_d} + m^2_{L_i} + m^2_{e_j} + |A^{ji}_e|^2)$$

$$- \frac{3}{10}g^2_1 \text{Tr}\{Ym^2\} - \frac{3}{5}g^2_1 M^2_1 - 3g^2_2 M^2_2\ ],$$

$$\frac{dm^2_{d_i}}{dt} = \frac{1}{8\pi^2}[\sum_j 2|Y^{ij}_d|^2(m^2_{\phi_d} + m^2_{d_i} + m^2_{Q_j} + |A^{ij}_d|^2)$$

$$+ \frac{1}{5}g^2_1 \text{Tr}\{Ym^2\} - \frac{4}{15}g^2_1 M^2_1 - \frac{16}{3}g^2_3 M^2_3\ ],$$

$$\frac{dm^2_{u_i}}{dt} = \frac{1}{8\pi^2}[\sum_j 2|Y^{ij}_u|^2(m^2_{\phi_u} + m^2_{u_i} + m^2_{Q_j} + |A^{ij}_u|^2)$$

$$- \frac{2}{5}g^2_1 \text{Tr}\{Ym^2\} - \frac{16}{15}g^2_1 M^2_1 - \frac{16}{3}g^2_3 M^2_3\ ],$$

$$\frac{dm^2_{Q_i}}{dt} = \frac{1}{8\pi^2}[\sum_{i,j}(|Y^{ji}_u|^2(m^2_{\phi_u} + m^2_{Q_i} + m^2_{u_j} + |A^{ji}_u|^2)$$

$$+ |Y^{ji}_d|^2(m^2_{\phi_d} + m^2_{Q_i} + m^2_{d_j} + |A^{ji}_d|^2))$$

$$+ \frac{1}{10}g^2_1 \text{Tr}\{Ym^2\} - \frac{1}{15}g^2_1 M^2_1 - 3g^2_2 M^2_2 - \frac{16}{3}g^2_3 M^2_3\ ],$$

$$\frac{dB}{dt} = \frac{1}{8\pi^2}[\ 3A^{ij}_u|Y^{ij}_u|^2 + 3A^{ij}_d|Y^{ij}_d|^2 + A^{ij}_e|Y^{ij}_e|^2 - \frac{3}{5}g^2_1 M_1 - 3g^2_2 M_2\ ].$$

$$(C.14)$$

where, as in Ref. 80, sums are implied over all indices not appearing on the left hand side and where

$$\text{Tr}\{Ym^2\} = \sum_{i=1}^{n_g}(m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{e_i}^2) \, . \qquad (\text{C.15})$$

The gaugino masses evolve as follows

$$\frac{d\text{ln}M_l}{dt} = -\frac{1}{8\pi^2}b_l g_l^2 \, . \qquad (\text{C.16})$$

APPENDIX D
THE β FUNCTION OF THE VEV

Here one examines under what conditions the β function of the vev agrees with that of the anomalous dimension of the corresponding scalar filed by considering a simple toy model. Spontaneously broken scalar electrodynamics is considered. The following bare Lagrangian describes the theory

$$\mathcal{L}_0 = -\frac{1}{4}F_{0\mu\nu}F_0^{\mu\nu} + |D\Phi_0|^2 - V(\Phi_0) + \mathcal{L}_{GF0} + \mathcal{L}_{ghost0} , \qquad (D.1)$$

where $D_\mu = \partial_\mu - ie_0\mathbf{Q}A_{0\mu}$ is the covariant derivative. $\Phi_0 = (\phi_0 + i\chi_0)/\sqrt{2}$ is a complex scalar field of charge one ($\mathbf{Q}\Phi_0 = \Phi_0$). The form of its potential is

$$V(\Phi_0) = \lambda_0(\Phi_0^\dagger\Phi_0 - \frac{1}{2}f_0^2)^2 , \qquad (D.2)$$

where f_0 is the minimum of the bare potential. If v_0 represents the bare vacuum expectation value of the scalar field, Φ_0, then to expand around the true vacuum the identification is made, $\Phi_0 = \Phi_0' + v_0$, where $< \Phi_0' >= 0$. If the primes are dropped, this transformation amounts to taking

$$\phi_0 \rightarrow \phi_0 + v_0 ,$$

$$\chi_0 \rightarrow \chi_0 . \qquad (D.3)$$

In terms of these, the gauge fixing (GF) and ghost Lagrangians are given by

$$\begin{aligned}\mathcal{L}_{GF0} &= -\frac{1}{2\xi_0}(\partial \cdot A_0 + e_0v_0\xi_0\chi_0)^2 \\ &= -\frac{1}{2\xi_0}(\partial \cdot A_0)^2 + e_0v_0A_0 \cdot \partial\chi_0 - \frac{1}{2}e_0^2v_0^2\xi_0\chi_0^2\end{aligned} \qquad (D.4)$$

and

$$\mathcal{L}_{ghost0} = \overline{\eta}_0(\frac{\delta F}{\delta\alpha})\eta_0 , \qquad (D.5)$$

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where $F = \partial \cdot A_0 + e_0 v_0 \xi_0 \chi_0$ is the gauge constraint, α is the gauge function, and η_0 is the ghost field. Under a gauge transformation

$$\delta A_{0\mu} = -\partial_\mu \alpha \ ,$$

$$\delta \phi_0 = e_0 \alpha \chi_0 \ , \tag{D.6}$$

$$\delta \chi_0 = -e_0 \alpha (\phi_0 + v_0) \ .$$

Therefore

$$\delta F = -\Box \alpha - e_0^2 v_0 \xi_0 (\phi_0 + v_0) \alpha \ ,$$

$$\frac{\delta F}{\delta \alpha} = -\Box - e_0^2 v_0 \xi_0 (\phi_0 + v_0) \ . \tag{D.7}$$

Consequently the ghost Lagrangian is

$$\bar{\eta}_0 \left(\frac{\delta F}{\delta \alpha} \right) \eta_0 = \partial_\mu \bar{\eta}_0 \partial^\mu \eta_0 - \xi_0 e_0^2 v_0^2 \bar{\eta}_0 \eta_0 - e_0^2 v_0 \xi_0 \bar{\eta}_0 \eta_0 \phi_0 \tag{D.8}$$

In terms of ϕ_0 and χ_0 the covariant derivative term above can be rewritten

$$\begin{aligned}
|D\phi_0|^2 &= \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 + \frac{1}{2} \partial_\mu \chi_0 \partial^\mu \chi_0 + e_0 A_{0\mu} (\chi_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \chi_0) \\
&\quad + \frac{1}{2} e_0^2 \phi_0^2 A_0 \cdot A_0 + \frac{1}{2} e_0^2 \chi_0^2 A_0 \cdot A_0 \\
&\quad + \frac{1}{2} e_0^2 v_0^2 A_0 \cdot A_0 + e_0^2 v_0 \phi_0 A_0 \cdot A_0 - e_0 v_0 A_0 \cdot \partial \chi_0
\end{aligned} \tag{D.9}$$

The potential becomes

$$\begin{aligned}
V &= \frac{1}{4} \lambda_0 \phi_0^4 + \lambda_0 v_0 \phi_0^3 + \frac{1}{2} \lambda_0 (3 v_0^2 - f_0^2) \phi_0^2 + \lambda_0 v_0 (v_0^2 - f_0^2) \phi_0 \\
&\quad + \frac{1}{4} \lambda_0 \chi_0^4 + \frac{1}{2} \lambda_0 (v_0^2 - f_0^2) \chi_0^2 \\
&\quad + \frac{1}{2} \lambda_0 \phi_0^2 \chi_0^2 + \lambda_0 v_0 \phi_0 \chi_0^2
\end{aligned} \tag{D.10}$$

The following are some of the relations amongst the bare and renormalized parameters that will be needed later

$$\begin{aligned}
\phi_0 &= Z_\phi^{\frac{1}{2}} \phi \ , \\
\lambda_0 &= \frac{Z_\lambda}{Z_\phi^2} \lambda \ , \\
v_0 &= Z_v^{\frac{1}{2}} v \ , \\
f_0 &= Z_f^{\frac{1}{2}} f \ ,
\end{aligned} \tag{D.11}$$

with

$$Z_\phi = 1 + \delta Z_\phi \, ,$$

$$Z_\lambda = 1 + \delta Z_\lambda \, ,$$

$$Z_v = 1 + \delta Z_v \, ,$$

$$Z_f = 1 + \delta Z_f \, .$$

(D.12)

The δZ terms are considered small and all calculations are done to first order in these. Furthermore, the following relations between the bare and renormalized n-point functions will be useful later

$$\Gamma_\phi^{(1)} = Z_\phi^{\frac{1}{2}}[-\lambda_0 v_0(v_0^2 - f_0^2) + T_0] \, ,$$

$$\Gamma_\phi^{(2)} = Z_\phi[p^2 - \lambda_0(3v_0^2 - f_0^2) + \Sigma_0(p^2)] \, ,$$

$$\Gamma_\phi^{(3)} = Z_\phi^{\frac{3}{2}}[-6\lambda_0 v_0 + \Theta_0] \, ,$$

$$\Gamma_\phi^{(4)} = Z_\phi^2[-6\lambda_0 + \Lambda_0] \, ,$$

(D.13)

where T_0, $\Sigma_0(p^2)$, Θ_0, and Λ_0 are the connected loop contributions to the one, two, three, and four point functions, respectively. From above the following identifications are made

$$M_0^2 = e_0^2 v_0^2 \, ,$$

$$m_0^2 = \lambda_0(3v_0^2 - f_0^2) \, ,$$

$$m_{\chi 0}^2 = \lambda_0(v_0^2 - f_0^2) + \xi_0 M_0^2 \, ,$$

$$m_{\eta 0}^2 = \xi_0 M_0^2 \, ,$$

(D.14)

where M_0, m_0, $m_{\chi 0}$, and $m_{\eta 0}$ are the masses of the gauge boson, Higgs boson, Nambu-Goldstone boson, and ghost, respectively. Making the choice $v = f$ (Note: This is true at one renormalization mass scale only) and dropping the

ϕ subscript, the first of Eq. (D.13) becomes

$$\Gamma^{(1)} = Z_\phi^{\frac{1}{2}} [-\lambda v \frac{Z_\lambda Z_{\tilde{v}}^{\frac{1}{2}}}{Z_\phi^2}(v^2 Z_v - f^2 Z_f) + T_0] ,$$

$$= Z_\phi^{\frac{1}{2}} [-\lambda v^3 \frac{Z_\lambda Z_{\tilde{v}}^{\frac{1}{2}}}{Z_\phi^2}(\delta Z_v - \delta Z_f) + T_0] , \tag{D.15}$$

$$= -\lambda v^3 \frac{Z_\lambda Z_{\tilde{v}}^{\frac{1}{2}}}{Z_\phi^{\frac{3}{2}}}(\delta Z_v - \delta Z_f) + T ,$$

$$= -\lambda v^3 (\delta Z_v - \delta Z_f) + T .$$

This leads to the result

$$\delta Z_v - \delta Z_f = \frac{T}{\lambda v^3} . \tag{D.16}$$

Similarly for the second of Eq. (D.13) it follows that

$$\Gamma^{(2)} = Z_\phi [p^2 - \lambda \frac{Z_\lambda}{Z_\phi^2}(3v^2 Z_v - f^2 Z_f) + \Sigma_0(p^2)] ,$$

$$= Z_\phi [p^2 - 2\lambda v^2 \frac{Z_\lambda}{Z_\phi^2}(1 + \frac{3}{2}\delta Z_v - \frac{1}{2}\delta Z_f) + \Sigma_0(p^2)] ,$$

$$= Z_\phi p^2 - m^2 \frac{Z_\lambda}{Z_\phi}(1 + \frac{3}{2}\delta Z_v - \frac{1}{2}\delta Z_f) + \Sigma(p^2) ,$$

$$= p^2 - m^2 + \delta Z_\phi p^2 - m^2(\delta Z_\lambda - \delta Z_\phi + \frac{3}{2}\delta Z_v - \frac{1}{2}\delta Z_f) + \Sigma(p^2) . \tag{D.17}$$

Rewriting $\Sigma(p^2) = \Sigma(0) + \Sigma'(0)p^2$, where the prime denotes differentiation with respect to p^2, Eq. (D.17) implies

$$\delta Z_\phi = -\Sigma'(0) \tag{D.18}$$

and

$$\delta Z_\lambda - \delta Z_\phi + \frac{3}{2}\delta Z_v - \frac{1}{2}\delta Z_f = \frac{\Sigma(0)}{m^2} \tag{D.19}$$

Above the identification has been made

$$m^2 = 2\lambda v^2 \tag{D.20}$$

for the renormalized mass of the Higgs boson. Now for the third of Eq. (D.13)

$$\Gamma^{(3)} = Z_\phi^{\frac{3}{2}}[\, -6\lambda v \frac{Z_\lambda Z_v^{\frac{1}{2}}}{Z_\phi^2} + \Theta_0\,]\ ,$$

$$= -6\lambda v \frac{Z_\lambda Z_v^{\frac{1}{2}}}{Z_\phi^{\frac{1}{2}}} + \Theta\ , \tag{D.21}$$

$$= -6\lambda v - 6\lambda v(\delta Z_\lambda + \frac{1}{2}\delta Z_v - \frac{1}{2}\delta Z_\phi) + \Theta\ ,$$

From Eq. (D.21) the following result is obtained

$$\delta Z_\lambda + \frac{1}{2}(\delta Z_v - \delta Z_\phi) = \frac{\Theta}{6\lambda v}\ . \tag{D.22}$$

Finally the fourth of Eq. (D.13) gives

$$\Gamma^{(4)} = Z_\phi^2[\, -6\lambda \frac{Z_\lambda}{Z_\phi^2} + \Lambda_0\,]\ ,$$

$$= -6\lambda Z_\lambda + \Lambda\ , \tag{D.23}$$

$$= -6\lambda - 6\lambda \delta Z_\lambda + \Lambda\ ,$$

from which follows the identification

$$\delta Z_\lambda = \frac{\Lambda}{6\lambda}\ . \tag{D.24}$$

The symbols T, $\Sigma(0)$, $\Sigma'(0)$, Θ, and Λ represent the renormalized loop contributions to the various n-point functions. They represent $\mathcal{O}(\lambda^2, \lambda e^2, e^4)$ corrections to the tree level results. They are correctly calculated in terms of renormalized quantities to the order that is being employed. Consequently the following renormalized masses will be needed

$$M^2 = e^2 v^2\ ,$$

$$m_\chi^2 = \xi M^2\ . \tag{D.25}$$

$$m_\eta^2 = \xi M^2\ .$$

The various bare propagators required to perform the calculations are

(a) the gauge boson : $\quad \dfrac{-i}{k^2 - M_0^2} [\, g_{\mu\nu} + (\xi_0 - 1) \dfrac{k_\mu k_\nu}{k^2 - \xi_0 M_0^2} \,]\,,$

(b) the Higgs boson : $\quad \dfrac{i}{k^2 - m_0^2}\,,$

(c) the Nambu $-$ Goldstone boson : $\quad \dfrac{i}{k^2 - m_{\chi 0}^2}\,,$ $\qquad$ (D.26)

(d) the ghost : $\quad \dfrac{-i}{k^2 - m_{\eta 0}^2}\,.$

The renormalized propagators are arrived at by simply substituting in them the proper renormalized masses.

The bare couplings are

(i) H-NG-γ : $\qquad\qquad\qquad e_0(p_1^{(in)} - p_2^{(out)})\,,$

(ii) 2H-2γ : $\qquad\qquad\qquad \frac{i}{2}e_0^2\, g_{\mu\nu}\,,$

(iii) 2NG-2γ: $\qquad\qquad\qquad \frac{i}{2}e_0^2\, g_{\mu\nu}\,,$

(iv) H-2γ : $\qquad\qquad\qquad ie_0^2 v_0\, g_{\mu\nu}\,,$

(v) 4H : $\qquad\qquad\qquad -\frac{i}{4}\lambda_0\,,$

(vi) 3H : $\qquad\qquad\qquad -i\lambda_0 v_0\,,$

(vii) 1H : $\qquad\qquad\qquad -i\lambda_0 v_0(v_0^2 - f_0^2)\,.$

(viii) 4NG : $\qquad\qquad\qquad -\frac{i}{4}\lambda_0\,,$

(ix) 2H-2NG : $\qquad\qquad\qquad -\frac{i}{2}\lambda_0\,,$

(x) H-2NG : $\qquad\qquad\qquad -i\lambda_0 v_0\,,$

(xi) H-2G : $\qquad\qquad\qquad i\xi_0 e_0^2 v_0\,,$ $\qquad$ (D.27)

where H, NG, γ, and G stand for Higgs, Nambu-Goldstone, photon, and ghost lines, respectively.

The results of the calculations are

$$\frac{\Sigma(0)}{m^2} = (13\lambda + \frac{1}{2}\xi e^2 + \frac{9}{2}\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon}$$

$$\Sigma'(0) = -e^2(3 - \xi)\frac{1}{16\pi^2\epsilon}$$

$$\frac{T}{\lambda v^3} = (6\lambda + \xi e^2 + 3\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon} \qquad \text{(D.28)}$$

$$\frac{\Theta}{6\lambda v} = (10\lambda - \xi e^2 + 3\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon}$$

$$\frac{\Lambda}{6\lambda} = (10\lambda - 2\xi e^2 + 3\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon}$$

Using Eqs. (D.16), (D.18), (D.19), (D.22), and (D.24) the following result is obtained for the counter terms

$$\delta Z_\phi = e^2(3 - \xi)\frac{1}{16\pi^2\epsilon}$$

$$\delta Z_\lambda = (10\lambda - 2\xi e^2 + 3\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon}$$

$$\delta Z_v = e^2(3 + \xi)\frac{1}{16\pi^2\epsilon} \qquad \text{(D.29)}$$

$$\delta Z_f = (-6\lambda + 3e^2 - 3\frac{e^4}{\lambda})\frac{1}{16\pi^2\epsilon}$$

Since the β function comes from these pole terms [81], it is evident that $\beta_v = \beta_\phi$ if $\xi = 0$, the Landau gauge.

APPENDIX E
NUMERICAL TECHNIQUES

The Runge-Kutta method is used to numerically integrate the β functions. There are eighteen coupled first order differential equations involved in running the Standard Model couplings. At one loop some equations decouple from the rest. For example, the gauge couplings are decoupled at one loop. However, the Yukawa β functions depend on both gauge couplings and Yukawas even at one loop as in Eqs. (A.8)-(A.9). One is working with two-loop β functions and these are all coupled. Standard Runge-Kutta programs are readily available; however, these usually assume knowledge of initial values of all couplings at the same scale t_o, where t will denote the scaling parameter. This presents somewhat of a problem since different couplings may be known at significantly different scales.

The method employed to solve the initialization problem is often referred to as "shooting" [82]. Simply put, a guess is made for the initial values of all the parameters at the chosen scale t_o. The parameters are then evolved to the various scales where one has known values, and the merits of the guess are assessed. The procedure is optimized, and one thereby arrives at a solution. Arriving at initial values of the couplings at the same initial scale therefore involves the solution of a possible eighteen nonlinear equations in eighteen unknowns. That is, in the most general case, if it is assumed that none of parameters are known at the desired initial scale t_o, then eighteen coupled

functions of eighteen variables can be defined as follows

$$F_i(\mathbf{x}(t_o)) = RK\{\mathbf{x}(t_o); t_i\}_i - X_i , \qquad (E.1)$$

where $\mathbf{x}(t_o)$ is an eighteen component vector denoting the unknown parameters at t_o, X_i is the known (possibly experimentally determined) value of the ith parameter at some scale t_i, and $RK\{\mathbf{x}(t_o); t_i\}_i$ is the value of the ith parameter resulting from numerically integrating the β evolution equations to a scale t_i given initial values, $\mathbf{x}(t_o)$ at t_o. Solution routines that solve N simultaneous nonlinear equations in N unknowns, that is, they solve

$$F_i(\mathbf{x}(t_o)) = 0 , \qquad (E.2)$$

are then used to find $\mathbf{x}(t_o)$. In the present case $x_i(t_o)$, $i = 1, ..., 18$, represent the eighteen parameters of the Standard Model, and the initial scale $\mu_o = e^{t_o} = M_Z$ has been chosen. The task has been appreciably simplified since one has most initial data at M_Z. Only the fermion masses are taken at a different scale. This reduces the number of simultaneous equations and therefore the computing task.

As the top quark and Higgs boson masses are unknown in the Standard Model at present, in the process of the analyses one is free to choose values for these masses at M_Z and then proceed to study the consequences. In other models in which there may be certain constraints ($e.g.$, in some GUTs the b and τ Yukawa couplings are equal at the scale of grand unification), one may incorporate these constraints into the functions (E.2). The freedom to choose a value for an unknown parameter may be replaced by such a constraint, and this may result in a definite prediction for that parameter. Constraints from grand unification and supersymmetry were used in Ref. 3 to arrive at possible values for the top quark and Higgs boson masses.

After all data is obtained at M_Z by employing the initialization procedure described above, the Runge-Kutta routines are used to evolve the parameters to any mass scale μ. Several figures are provided in Section 3.7 displaying the evolution of many of the parameters of the Standard Model for the case $m_t(M_Z) = 95$ GeV. As discussed in Section 3.4, given the evolution of the running parameters m_t and λ, the physical top quark and Higgs boson masses may be found by solving Eqs. (3.4.1) and (3.4.2), respectively. These equations are solved using the nonlinear equation solution routines described above in the initialization procedure. For example, consider the above situation in which $m_t(M_Z) = 95$ GeV, and suppose one wishes to find the corresponding physical mass M_t in this case. The solution algorithm may be described as follows: A guess is made for M_t, then the running parameters are evolved using Runge-Kutta to this mass scale (i.e., $\mu = M_t$). The guess is tolerated depending on how accurately one wishes Eq. (3.4.1) to be satisfied when values for $\alpha_s(M_t)$ and $m_t(M_t)$ are substituted. The solution routines effectively optimize this shooting or guessing procedure and yield a value for M_t corresponding to the value $m_t(M_Z)$. In this particular case, the result is $M_t = 100$ GeV.

APPENDIX F
EXPLICIT FROM OF $\delta(\mu)$

In Ref. 34 the radiative corrections term $\delta(\mu)$, from Eq. (3.4.2), is derived. In this appendix, its explicit form is presented as it appears in this reference except for some minor notational changes. In the following, s and c refer to $\sin\theta_W$ and $\cos\theta_W$, respectively. Also, ξ is defined to be the ratio M_H^2/M_Z^2.

$$\delta(\mu) = \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{8\pi^2} \{\xi f_1(\xi,\mu) + f_0(\xi,\mu) + \xi^{-1} f_{-1}(\xi,\mu)\} , \tag{F.1}$$

where the various functions are defined as follows

$$f_1(\xi,\mu) = 6\ln\frac{\mu^2}{M_H^2} + \frac{3}{2}\ln\xi - \frac{1}{2}Z(\frac{1}{\xi}) - Z(\frac{c^2}{\xi}) - \ln c^2 + \frac{9}{2}(\frac{25}{9} - \frac{\pi}{\sqrt{3}}) ,$$

$$f_0(\xi,\mu) = -6\ln\frac{\mu^2}{M_Z^2}[1 + 2c^2 - 2\frac{M_t^2}{M_Z^2}] + \frac{3c^2\xi}{\xi - c^2}\ln\frac{\xi}{c^2} + 2Z(\frac{1}{\xi})$$

$$+ 4c^2 Z(\frac{c^2}{\xi}) + \frac{3c^2\ln c^2}{s^2} + 12c^2\ln c^2 - \frac{15}{2}(1 + 2c^2)$$

$$- 3\frac{M_t^2}{M_Z^2}[2Z(\frac{M_t^2}{M_Z^2\xi}) + 4\ln\frac{M_t^2}{M_Z^2} - 5] , \tag{F.2}$$

$$f_{-1}(\xi,\mu) = 6\ln\frac{\mu^2}{M_Z^2}[1 + 2c^4 - 4\frac{M_t^4}{M_Z^4}] - 6Z(\frac{1}{\xi}) - 12c^4 Z(\frac{c^2}{\xi}) - 12c^4\ln c^2$$

$$+ 8(1 + 2c^4) + 24\frac{M_t^4}{M_Z^4}[\ln\frac{M_t^2}{M_Z^2} - 2 + Z(\frac{M_t^2}{M_Z^2\xi})] ,$$

with

$$Z(z) = \begin{cases} 2A\tan^{-1}(1/A) , & (z > \frac{1}{4}) \\ A\ln[(1 + A)/(1 - A)] , & (z < \frac{1}{4}) \end{cases} , \tag{F.3}$$

$$A \equiv |1 - 4z|^{\frac{1}{2}} .$$

APPENDIX G
'METAPLECTONS'

G.1 Introduction

Theories of objects with arbitrary spin and statistics known as anyons are possible in $(2 + 1)$-dimensions [83]. Such theories have been applied successfully in condensed matter physics. For example, they have been used in the description of quasiparticles in the fractional quantum Hall effect. Some of these theories employ a topological quantity known as the Chern-Simons term. This term is sometimes generated radiatively through the use of heavy fermions rather than put in by hand. This work will consider such theories. The statistics of the anyons in these theories will depend on the coefficient of the Chern-Simons term which subsequently will be seen to depend on the number of fermions and on their respective charges. If all the fermions carry the same fundamental charge, then by varying the number of fermions a certain "spectrum" of fractional statistics is possible. This spectrum is, however, limited in its extent, and the question may be raised as to whether it can be broadened by effecting changes or extensions to the fermion sector. This work addresses this question. The answer is yes, and the change or extension to the model considered here will involve fields in a spinor or metaplectic representation of the orthosymplectic Lie supergroup. The representations are infinite dimensional but have graded dimension that is finite and can be fractional.

In Section G.2, a brief introduction to anyons is given as well as an account of the role of the Chern-Simons term in such theories. The specific model being considered is presented, and the extension of the fermion sector is motivated. Section G.3 contains a self-contained exposition of the orthosymplectic Lie supergroup and its spinor (or singleton) representations. The oscillator construction is discussed and used to arrive at the metaplectic representations. Section G.4 applies the results of Section G.3 in extending the model of Section G.2. The "new" theory has a broader spectrum. The statistics are tuned in the fermion sector now extended by the "metaplectons". A short summary of the results of the appendix and some remarks make up the contents of Section G.5.

G.2 Anyons

In $(2 + 1)$-dimensions, the familiar Bose and Fermi statistics are special cases of more general fractional statistics. Under the interchange of identical particles, the wave function changes by an arbitrary phase rather than just being symmetric or antisymmetric as in $3 + 1$ dimensions. An anyon is any $(2 + 1)$-dimensional object with fractional statistics. Such an object can be viewed as a composite consisting of a "bare" Bose or Fermi point charge bound to an infinitely thin flux tube. Interchanging two of these identical composites yields any phase depending on how the fluxes are tuned. The phase is, in fact, an Aharanov-Bohm phase resulting from the long range interaction of one composite's charge with the other composite's flux tube. A particle of charge, Q, encircling an infinitely thin flux (Φ) tube picks up an Aharanov-Bohm phase, $e^{i\theta_{AB}}$, where $\theta_{AB} = Q\Phi$ [84].

A way of implementing fractional statistics in certain $(2 + 1)$ models is through the use of an Abelian Chern-Simons term. A Lagrangian, $\mathcal{L}$, with a conserved current, j^μ, can have additional gauge invariant interactions appended to it [85]

$$\Delta\mathcal{L} = -j^\mu A_\mu + \frac{1}{2}\alpha\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho , \qquad (G.2.1)$$

where A_μ is the so-called statistics gauge field and has dimension of $[mass]^{\frac{1}{2}}$, and α is the Chern-Simons coupling with dimension of $[mass]$. The equation of motion of this gauge field gives,

$$j^\mu = \alpha\epsilon^{\mu\nu\rho}\partial_\nu A_\rho . \qquad (G.2.2)$$

Integrating the zeroth component of this equation over all space yields

$$\int d^2x\, j^o = \alpha \int d^2x\, B , \quad where \quad B = \epsilon^{ij}\partial_i A_j ,$$
$$Q = \alpha\Phi . \qquad (G.2.3)$$

Hence charged particles carry statistical flux equal to Q/α. This result is independent of the insertion of a kinetic term for the statistics gauge field

$$\mathcal{L}_{kin} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \; , \tag{G.2.4}$$

where

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \; . \tag{G.2.5}$$

As seen in the following argument, if a kinetic term were present, the equation of motion would be altered to

$$j^{\mu} = \alpha \epsilon^{\mu\nu\rho} \partial_{\nu} A_{\rho} + \partial_{\nu} F^{\mu\nu} \; . \tag{G.2.6}$$

The "statistics" photon in the quantized theory will have a mass, α, due to the Chern-Simons term; α will act as a long range cut-off for the electric and magnetic fields [86]. Therefore the integral over all space of the time component of Eq. (G.2.6) will still give Eq. (G.2.3) since $\int d^2x \nabla \cdot \vec{E}$ vanishes.

In a Lagrangian with Eq. (G.2.1) added to it, an effective current, j^{μ}_{eff}, can be defined as the coefficient of $\partial_{\mu}\chi$ under the gauge transformation $A_{\mu} \rightarrow A_{\mu} - \partial_{\mu}\chi$ [87]

$$j^{\mu}_{eff} = j^{\mu} - \frac{1}{2} \alpha \epsilon^{\mu\nu\rho} \partial_{\nu} A_{\rho} \; . \tag{G.2.7}$$

Making use of the equation of motion, Eq. (G.2.2), results in

$$j^{\mu}_{eff} = \frac{1}{2} j^{\mu} \; . \tag{G.2.8}$$

Accordingly, this leads to an effective charge in terms of the physical charge, Q,

$$Q_{eff} = \frac{1}{2} Q \; . \tag{G.2.9}$$

In this way an anyon arising from a Chern-Simons term can be considered as point-particle-flux-tube composite with statistical charge, Q_{eff}, and flux, Φ.

If two such objects are adiabatically interchanged by moving each one half way around the other, the Aharanov-Bohm interchange phase induced in the wave function is

$$\theta_{AB} = \frac{1}{2}Q_{eff}\Phi + \frac{1}{2}Q_{eff}\Phi$$
$$= Q_{eff}\Phi \ . \tag{G.2.10}$$

Inserting the results of Eqs. (G.2.3) and (G.2.9) above yields

$$\theta_{AB} = \frac{Q^2}{2\alpha} \ . \tag{G.2.11}$$

The full quantum statistics phase is arrived at by also including the bare statistics phase (θ_{bare})

$$\theta = \theta_{bare} + \theta_{AB} \ . \tag{G.2.12}$$

Consider the following functional for the statistics gauge field, A_μ, in $(2+1)$-dimensions involving charge, e (which has dimension of $[mass]^{\frac{1}{2}}$), massive fermions (which have dimension of $[mass]$)

$$\Delta[A] = \int D\psi D\overline{\psi} \, \exp\{i \int d^3x \, \overline{\psi}\gamma^\mu(i\partial_\mu - eA_\mu - m)\psi\}$$
$$= \det(i\not{\mathcal{D}} - m) \ , \tag{G.2.13}$$

where $\not{\mathcal{D}} = \gamma^\mu(\partial_\mu + ieA_\mu)$. Through vacuum polarization a Chern-Simons term is induced to lowest order in the momentum [88]

$$\ln\Delta[A] = i\frac{e^2}{8\pi}\frac{m}{|m|} \int d^3x \, \epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + I[A] \ . \tag{G.2.14}$$

The higher order terms in $I[A]$ are proportional to powers of e and $1/m$, and these are assumed to satisfy the condition

$$m \gg e^2 \ . \tag{G.2.15}$$

Comparing expression (G.2.14) with the second term on the right hand side of Eq. (G.2.1) and assuming the fermion masses are positive yields the identification

$$\alpha = \frac{e^2}{4\pi} \ . \tag{G.2.16}$$

Note that the induced (Chern-Simons) term violates parity, a fact that is not surprising since a fermion mass term is parity violating in $(2+1)$-dimensions. Under parity, defined by

$$(t, x, y) \rightarrow (t, -x, y) \ ,$$

(the usual definition of parity, $(x, y) \rightarrow (-x, -y)$, is a rotation is this case) the fermion mass term transforms

$$m\overline{\psi}\psi \rightarrow -m\overline{\psi}\psi \ . \tag{G.2.17}$$

Suppose there had been N species of fermions with charges e_k, $k = 1, ..., N$, then

$$\begin{aligned} \Delta_N[A] &= \int D\psi D\overline{\psi} \, \exp\{i \int d^3x \sum_{k=1}^{N} \overline{\psi}_k(i\mathcal{P}_k - m)\psi_k\} \\ &= \prod_{k=1}^{N} \det(i\mathcal{P}_k - m) \ , \end{aligned} \tag{G.2.18}$$

where $\mathcal{P}_k = \gamma^\mu(\partial_\mu + ie_k A_\mu)$. The induced Chern-Simons term then yields the following identification for the coupling

$$\alpha = \frac{1}{4\pi} \sum_{k=1}^{N} e_k^2 \ . \tag{G.2.19}$$

Some models exhibiting fractional statistics and employing a Chern-Simons term sometimes replace it with a Lagrangian,

$$\mathcal{L}_N = \sum_{k=1}^{N} \overline{\psi}_k(i\not{\partial} - e\not{A} - m)\psi_k \ , \tag{G.2.20}$$

thereby generating the Chern-Simons term radiatively as previously discussed. This will yield a Chern-Simons coefficient given by Eq. (G.2.19) with $e_k = e$ for all $k = 1, ..., N$

$$\alpha = N\frac{e^2}{4\pi} \ . \tag{G.2.21}$$

Consider the following Lagrangian [89]

$$\mathcal{L} = |(\partial_\mu + iqeA_\mu)\phi|^2 - M^2|\phi|^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_N \ . \qquad \text{(G.2.22)}$$

The bare boson ($\theta_{bare} = 0$) field, ϕ, carries charge $Q = qe$ (q is an integer) and has mass, M. Interchanging two of these quanta will give Aharanov-Bohm phase (G.2.11). Hence their quantum statistics are given by

$$\theta = \frac{2\pi q^2}{N} \ . \qquad \text{(G.2.23)}$$

If, for example, $q = 1$ and $N = 2$, then the statistics are those of fermions ($\theta = \pi$). More exotic phases are possible with more fermions (i.e., $N \geq 3$). Indeed, for $q = 1$ and $N = 4$, the statistics are those of semions ($\theta = \pi/2$).

It is worth noting that a composite of N anyons has an interchange phase ($\theta_{AB}(N)$) that is N^2 times the interchange phase of one anyon [90]. Therefore in this case the full quantum statistics phase is

$$\begin{aligned}
\theta(N) &= \theta_{bare}(N) + N^2\theta_{AB} \\
&= \theta_{bare}(N) \bmod 2\pi \ .
\end{aligned} \qquad \text{(G.2.24)}$$

Hence such an object (an N-anyon) exhibits no fractional statistics but rather exhibits its bare statistics.

Evidently the Lagrangian (G.2.22) which has a global $U(N)$ symmetry cannot produce arbitrary fractional statistics for a given value of q, rather the statistics phases are constrained by the integer N through Eq. (G.2.23). One way of extending the range of θ is by having non-integer N. This may seem impossible if N is thought of as the number of fermions but not so impossible if it is identified with the power of the determinant (G.2.18). The purpose of this work is to point out that non-integer powers of this determinant are attainable by considering more exotic internal symmetry groups. These produce new

types of fractional statistics, in the sense discussed above, hitherto unknown in the literature. This is introduced through the use of supergroup representations with fractional dimensions.

G.3 The Metaplectic Representations of $OSp(r/2m, R)$

The use of supergroup representations as a device to yield fractional powers of determinants is not unknown in the literature [91]. The covariant quantization of the Green-Schwarz superstring appears to require the introduction of infinite sets of extra fields, of ghosts, and of ghosts for ghosts. It has been suggested that the superstring action can be written in an OSp invariant form for certain choices of gauge with the "infinite towers" of ghosts transforming as spinor (or metaplectic) representations of the $OSp(r/2m, R)$ Lie supergroups [92]. The harmonic oscillator method for constructing unitary lowest weight representations (i.e., representations in which one of the generators is bounded from below) of the noncompact supergroup, $OSp(2n/2m, R)$, has been presented in the literature [93]. In the following the reader is assumed to have some familiarity with Lie supergroups and superalgebras (for a review see reference [94]).

The orthosymplectic Lie supergroup, $OSp(2n/2m, R)$, has even subgroup, $SO(2n) \times Sp(2m, R)$. The number of even generators is $\frac{1}{2}(2n)(2n-1) + \frac{1}{2}(2m)(2m+1)$. The number of odd generators is $(2n)(2m)$. $OSp(2n/2m, R)$ has $U(m/n)$ as its maximal rank, compact subsupergroup. With respect to this subsupergroup, $OSp(2n/2m, R)$ admits a Jordan decomposition or three-grading

$$L = L_{-1} \oplus L_o \oplus L_{+1} .$$
<div align="right">(G.3.1)</div>

L and L_o denote the Lie superalgebras of the supergroups $OSp(2n/2m, R)$ and $U(m/n)$, respectively. $U(m/n)$ has even subgroup $U(n) \times U(m)$ with respect to which $SO(2n) \times Sp(2m, R)$ has a three-grading. The Lie superalgebra of $OSp(2n/2m, R)$ is realized as bilinears of a set of $f = 2p + \varepsilon$ superoscillators (and their hermitian conjugates)

$$\xi_A(s) = \begin{pmatrix} a_i(s) \\ \alpha_\mu(s) \end{pmatrix}, \quad \eta_A(s) = \begin{pmatrix} b_i(s) \\ \beta_\mu(s) \end{pmatrix}, \quad \zeta_A(s) = \begin{pmatrix} c_i \\ \gamma_\mu \end{pmatrix}$$

$$\xi^A(s) = \xi^\dagger_A(s) , \quad \eta^A(s) = \eta^\dagger_A(s) , \quad \zeta^A = \zeta^\dagger_A \tag{G.3.2}$$

$$i = 1, ..., m ; \quad \mu = 1, ..., n ; \quad s = 1, ..., p .$$

Depending on whether there is an even or odd number of superoscillators, ε will equal 0 or 1. The metaplectic (or singleton) representations correspond to $f = 1$ ($p = 0, \varepsilon = 1$). The first m components of the superoscillators are bosonic while the last n components are fermionic. The superoscillators ξ_A, η_A, ζ_A (ξ^A, η^A, ζ^A) transform in the covariant (contravariant) fundamental representation of $U(m/n)$ and satisfy the supercommutation relations

$$[\xi_A(s), \xi^B(t)\} = [\eta_A(s), \eta^B(t)\} = \delta^B_A \delta_{st} ,$$

$$[\zeta_A, \zeta^B\} = \delta^B_A . \tag{G.3.3}$$

All other supercommutators vanish. The graded commutator is defined as

$$[\xi_A, \xi^B\} = \xi_A \xi^B - (-1)^{g(A)g(B)} \xi^B \xi_A , \tag{G.3.4}$$

where $g(A)$ is 0 or 1 depending on the grade of the index $A = (i, \mu)$. L_{-1} consists of di-annihilation operators of the form

$$\vec{\xi}_A \cdot \vec{\eta}_B + \vec{\eta}_A \cdot \vec{\xi}_B + \varepsilon \, \zeta_A \zeta_B . \tag{G.3.5}$$

Correspondingly, L_{+1} consists of di-creation operators

$$\vec{\xi}^B \cdot \vec{\eta}^A + \vec{\eta}^B \cdot \vec{\xi}^A + \varepsilon \, \zeta^B \zeta^A \ . \tag{G.3.6}$$

The bilinears of L_o are of the form

$$\vec{\xi}^A \cdot \vec{\xi}_B + (-1)^{g(A)g(B)} \vec{\eta}_B \cdot \vec{\eta}^A + \frac{\varepsilon}{2} \, [\zeta^A \zeta_B + (-1)^{g(A)g(B)} \zeta_B \zeta^A] \ . \tag{G.3.7}$$

The vector notation employed above is defined as follows

$$\vec{\xi}_A \cdot \vec{\eta}_B \equiv \sum_{s=1}^{p} \xi_A(s) \, \eta_B(s) \ . \tag{G.3.8}$$

The unitary lowest weight representations of OSp are constructed by considering a set of states, $|\Omega\rangle$, in the (super) Fock space of superoscillators. These states transform irreducibly under L_o (the Lie algebra of the maximal compact subsupergroup) and are annihilated by the operators belonging to L_{-1}. Operating successively on $|\Omega\rangle$ with the elements of L_{+1}, a unitary lowest weight representation is generated

$$\{|\Omega\rangle \oplus L_{+1}|\Omega\rangle \oplus (L_{+1})^2|\Omega\rangle \oplus \cdots\} \ . \tag{G.3.9}$$

For $f = 1$, $|\Omega\rangle = \{|0\rangle, \zeta^A|0\rangle\}$ are the only two lowest weight states, where $|0\rangle$ is the Fock space vacuum annihilated by all annihilation operators.

Application of the oscillator construction to $SO(2n)$ or $Sp(2m, R)$ is straightforward. In the case of $SO(2n)$, the set of $f = 2p+\varepsilon$ fermionic oscillators would consist of

$$\alpha_\mu(s) \, , \quad \beta_\mu(s) \, , \quad \gamma_\mu$$

$$\alpha^\mu(s) = \alpha_\mu^\dagger(s) \, , \quad \beta^\mu(s) = \beta_\mu^\dagger(s) \, , \quad \gamma^\mu = \gamma_\mu^\dagger \tag{G.3.10}$$

$$\mu = 1, ..., n \ ; \quad s = 1, ..., p \ .$$

Clearly the form of the unitary irreducible lowest weight representation (Eq. (G.3.9)) indicates that it is finite dimensional when the oscillators are fermionic, as is the case for $SO(2n)$. For $Sp(2m, R)$, the oscillators are bosonic, and the representations are infinite dimensional. The unitary irreducible representations can be put in a form that displays the even subgroup $(SO(2n) \times Sp(2m, R))$ structure. The two irreducible spinor representations of $OSp(2n/2m, R)$ can then be written

$$\Psi_+ = \begin{pmatrix} \psi_+^{(0)} \\ \psi_-^{(1)} \\ \psi_+^{(2)} \\ \vdots \end{pmatrix}, \quad \Psi_- = \begin{pmatrix} \psi_-^{(0)} \\ \psi_+^{(1)} \\ \psi_-^{(2)} \\ \vdots \end{pmatrix}. \tag{G.3.11}$$

The ψ's are $SO(2n)$ chiral spinors with the Grassmann parity alternating down each "tower" (the Grassmann parity of the first in the sequence is arbitrary). The superscript indicates their transformation properties under $U(m)$, the maximal compact subgroup of $Sp(2m, R)$. For example, $\psi_+^{(k)} \equiv \psi_+^{i_1 \cdots i_k}$ is a (+)-chirality $SO(2n)$ spinor, Grassmann odd field transforming as a kth rank symmetric tensor of $SU(m)$. It is possible to combine the two irreducible representations to form a "Dirac" OSp spinor

$$\Psi = \begin{pmatrix} \Psi^{(0)} \\ \Psi^{(1)} \\ \Psi^{(2)} \\ \vdots \end{pmatrix}, \quad \Psi^{(n)} = \begin{pmatrix} \psi_+^{(n)} \\ \psi_-^{(n)} \end{pmatrix}. \tag{G.3.12}$$

The two irreducible ((+) and (−) chirality) spinors (in the basis that they have been presented above) can be recovered by applying the following projection operators

$$\mathbf{P}_\pm = \mathbf{1} \otimes P_\pm, \tag{G.3.13}$$

where $\mathbf{1}$ is the infinite unit matrix and $P_\pm$ are the $SO(2n)$ projectors

$$P_\pm = \frac{1}{2}(1 \pm \overline{\Gamma}), \quad \overline{\Gamma} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{G.3.14}$$

and $\overline{\Gamma}$ has been displayed in 2×2 block form. $SO(2n+1)$ has no such projectors and correspondingly $OSp(2n+1/2m, R)$ has only one irreducible singleton representation. Indeed, instead of a three-grading, $OSp(2n+1/2m, R)$ admits a five-grading (Kantor decomposition) with respect to its maximal compact subsupergroup [95]

$$L = L_{-1} \oplus L_{-\frac{1}{2}} \oplus L_o \oplus L_{+\frac{1}{2}} \oplus L_{+1} \ . \qquad (G.3.15)$$

The invariant inner product of complex OSp spinors is given by

$$\overline{\Psi}\Lambda \equiv \sum_{k=0}^{\infty} \overline{\Psi}^{i_1 \cdots i_k} \Lambda^{j_1 \cdots j_k} \delta_{i_1 j_1} \cdots \delta_{i_k j_k} \ , \qquad (G.3.16)$$

where Ψ and Λ are two OSp spinors, $\overline{\Psi} = \Psi^\dagger \Gamma_t$ is the Pauli adjoint and Γ_t represents the product of the time-like gamma matrices.

The graded dimension of a representation is defined as the number of Bose states minus the number of Fermi states in the vector space. It can be computed by evaluating the character for the identity matrix [96]. Consider an irreducible spinor representation of $OSp(r/2m, R)$. If $r = 2n$, then the dimension of an $SO(2n)$ chiral spinor is $d_{spinor}(SO(2n)) = 2^{n-1}$. If $r = 2n + 1$, then $d_{spinor}(SO(2n+1)) = 2^n$. The dimension of a kth rank symmetric tensor of $SU(m)$ is given by the following binomial coefficient

$$d_{sym.k-tensor}(SU(m)) = \binom{m+k-1}{m-1} \ . \qquad (G.3.17)$$

If each Grassmann odd field counts as -1 and each Grassmann even field counts as $+1$, then the following regulated sum of the $SU(m)$ symmetric tensors in Ψ yields

$$d_{spinor}(Sp(2m, R)) = \sum_{k=0}^{\infty} (-1)^k \binom{m+k-1}{m-1} \rightarrow \lim_{x \to 1} \sum_{k=0}^{\infty} (-1)^k x^k \binom{m+k-1}{m-1}$$
$$= \lim_{x \to 1} (1+x)^{-m}$$
$$= 2^{-m} \ .$$

$$(G.3.18)$$

The graded dimension of an $OSp(r/2m, R)$ spinor is therefore given by [97]

$$d_{meta}(OSp(r/2m, R)) = 2^{-m}d_{spinor}(SO(r)) . \tag{G.3.19}$$

G.4 New Representations of Anyons

The supergroup representations of the last section will now be used to show how to extend the statistics spectrum discussed previously. As an example, consider the following functional of A_μ

$$\Delta_{meta}[A] = \int \prod_{k=1}^{\infty} D\psi_k D\overline{\psi}_k \exp\{i \int d^3x \sum_{k=1}^{\infty} \overline{\psi}_k(i\partial - eA - m)\psi_k\} \tag{G.4.1}$$
$$= [\det(i\mathcal{D} - m)]^{(1-1+1-1+\cdots)} ,$$

where the ψ_k's are $(2+1)$ space-time spinors having alternating Grassmann parity with the odd subscript ones odd and the even subscript ones even. This sequence of fields $\{\psi_k\}, k = 1, 2, ...,$ can be fitted into a metaplectic representation of $OSp(2/2, R)$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \end{pmatrix} . \tag{G.4.2}$$

The ψ_k's are one component chiral $SO(2)$ spinors and are scalars under $Sp(2, R)$ (a mass term is possible for chiral $SO(2)$ spinors). The Lagrangian for these fields can be rewritten using the boldface notation introduced in Section G.3

$$\mathcal{L}_{meta} = \overline{\Psi}(i\mathcal{D} - m)\Psi \equiv \sum_{k=1}^{\infty} \overline{\psi}_k(i\mathcal{D} - m)\psi_k . \tag{G.4.3}$$

The alternating sum in the power of the determinant of Eq. (G.4.1) can be computed in a natural way using the group invariant introduced in Eq. (G.3.19),

$$d_{meta}(OSp(2/2, R)) = \frac{1}{2} . \tag{G.4.4}$$

Therefore,

$$\Delta_{meta}[A] = [\det(i\slashed{D}-m)]^{\frac{1}{2}} . \tag{G.4.5}$$

The induced Chern-Simons term will then have coupling,

$$\alpha = \frac{1}{2}\left(\frac{e^2}{4\pi}\right) . \tag{G.4.6}$$

The square root of the Dirac operator enters the analysis of the $SU(2)$ global anomaly in four dimensions [98]. There, integrating over an $SU(2)$ Weyl doublet, ψ, gives

$$\int D\psi D\overline{\psi} \, \exp\{i \int d^4x \, \overline{\psi}\slashed{D}\psi\} = [\det i\slashed{D}]^{\frac{1}{2}} . \tag{G.4.7}$$

Such a representation for the Weyl fermions is possible because $SU(2)$ has no local anomaly

$$Tr(\{T^a, T^b\}, T^c) = 0 , \tag{G.4.8}$$

where the T's are $SU(2)$ generators. In the present case no such condition need be satisfied by the gauge group. The nontriviality of the homotopy group, $\Pi_4(SU(2))$, is crucial in the analysis of the global anomaly. In three dimensions the relevant group is Π_3. However, $\Pi_3(U(1)) = 0$, and hence no global anomaly plagues the metaplecton theory being considered here. Clearly metaplecton theories involving non-Abelian gauge groups with nontrivial homotopy groups may suffer from such problems.

Using the result of Eq. (G.4.6), the Lagrangian,

$$\mathcal{L} = |(\partial_\mu + iqeA_\mu)\phi|^2 - M^2|\phi|^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{meta} , \tag{G.4.9}$$

will lead to a statistics phase,

$$\theta = 4\pi q^2 , \tag{G.4.10}$$

for the charge qe excitations; in other words, they are bosons. It appears that the $OSp(2/2, R)$ global symmetry has alone, at least, not produced any different statistics (in the sense of Section G.2). However, consider the Lagrangian,

$$\mathcal{L}' = |(\partial_\mu + iqeA_\mu)\phi|^2 - M^2|\phi|^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{meta} + \mathcal{L}_{N'} , \qquad (G.4.11)$$

where

$$\mathcal{L}_{N'} = \sum_{k=1}^{N'} \overline{\psi}_k(i\partial\!\!\!/ - e\!\!\!/A - m)\psi_k . \qquad (G.4.12)$$

This model has a $U(N')\times OSp(2/2, R)$ global symmetry and will lead to Chern-Simons coupling,

$$\alpha = \frac{2N' + 1}{2}(\frac{e^2}{4\pi}) , \qquad (G.4.13)$$

and to statistics phase,

$$\theta = \frac{4\pi q^2}{2N' + 1} . \qquad (G.4.14)$$

This phase (taking $N' > 1$) cannot be achieved in model (G.2.22) for any value of N (N an integer). So the goal of attaining a wider range of fractional statistics has been reached through the addition of $\mathcal{L}_{meta}$. More phases outside the range of (G.2.22) are possible by generalizing to $OSp(2/2m, R)$, with the fields in $\mathcal{L}_{meta}$ transforming as the metaplectic representation of this supergroup. The statistics phase of a $U(N') \times OSp(2/2m, R)$ model would be

$$\theta = \frac{2\pi q^2}{N' + d_{meta}(m)} , \qquad (G.4.15)$$

where (using Eq. (G.3.19))

$$d_{meta}(m) \equiv d_{meta}(OSp(2/2m, R)) = 2^{-m} . \qquad (G.4.16)$$

It should be emphasized that the statement that Eq. (G.4.14), associated with Lagrangian (G.4.11), gives phases outside the range of (G.2.22), is a valid one for fixed q in (G.2.22) and (G.4.11). Indeed statistics (G.4.15) with given

values of q, N', and m are attainable using (G.2.22) with no OSp symmetry. However, the ϕ field would then have statistics charge, q', such that

$$q' = \begin{cases} 2^{\frac{m}{2}}q \, , & m \text{ even} \; ; \\ 2^{\frac{m+1}{2}}q \, , & m \text{ odd} \, , \end{cases} \tag{G.4.17}$$

and there would have to be more than N' fermions

$$N = \begin{cases} 2^m N' + 1 \, , & m \text{ even} \; ; \\ 2(2^m N' + 1) \, , & m \text{ odd} \, . \end{cases} \tag{G.4.18}$$

G.5 Conclusion

This appendix has shown how to generate a new class of models of anyons in $(2+1)$-dimensions employing a radiatively generated Chern-Simons term that have their "spectrum" of fractional statistics broadened without varying any of their parameters [11]. The fields introduced to induce the Chern-Simons term are supplemented with fields forming a metaplectic or spinor representation of the Lie supergroup, $OSp(r/2m, R)$. The representations are infinite dimensional, since the even Lie subgroup of $OSp(r/2m, R)$ is $SO(r) \times Sp(2m, R)$, and $Sp(2m, R)$ has infinite dimensional spinor representations. These metaplectic representations have the special property that they yield fractional powers of determinants when the fields are integrated out. In the models mentioned above they lead to statistics outside the range achievable with any number of ordinary fermions.

There are many ways to generate exotic statistics in $(2+1)$-dimensions. This work has presented a new method. In general the distinguishing feature between these methods is the fermion representation. As it stands, the new fermion representations introduced are indistinguishable from any other since the fermions are integrated out. In this regard the new representations are nothing more than a device. To distinguish two such equivalent theories, it

is necessary to either include more degrees of freedom such as scalars transforming nontrivially under the symmetry group or to gauge some parts of the group. Whether or not this will lead to any interesting results is not known. However, as it stands the model "forgets" about the metaplectons after they are integrated out. Gauging is a means of effectively leaving a "memory" of the OSp group after the metaplectons are integrated out.

This appendix has not dealt with the dynamics of anyons but rather with an alternate means of inducing fractional statistics. It is noted that questions of this sort have been addressed elsewhere, and a relativistic wave equation for anyons has been formulated derivable from an appropriate action [99].

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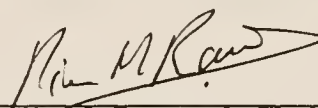
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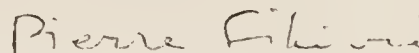
BIOGRAPHICAL SKETCH

The author was born in Cuba on June 25, 1961. His family was exiled and he arrived in this country on January 31, 1968. His physics career effectively began that year. In elementary school, after reading books on special relativity and Albert Einstein, he decided he wanted to be a theoretical physicist. He graduated from the University of Miami in May 1984 with a B.S. in physics and mathematics. Later, in June 1987, he received his M.S. in physics from that same institution. That summer he married Maria Fernandez and entered the University of Florida, eventually to work with Pierre Ramond on theoretical particle physics. The author will be leaving the University of Florida with a Ph.D. in theoretical physics and will be moving to Boston where he will hold a postdoctoral position with the theoretical physics group at M.I.T.

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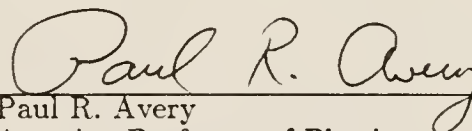
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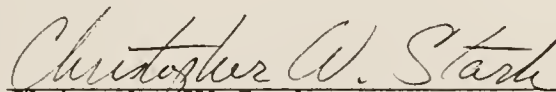
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Paul R. Avery
AssociateProfessor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Christopher W. Stark
AssociateProfessor of Mathematics

This dissertation was submitted to the Graduate Faculty of the Department of Physics in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

May 1993

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Dean, Graduate school

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